The Logic of (Where and) While in the 13th and 14th Centuries

Sara L. Uckelman
s.l.uckelman@durham.ac.uk
@SaraLUckelman

25 September 2018

A bit of background

- What is medieval logic?
- Why study it?

Logic in the European Middle Ages

How many of you can name a medieval logician?

Logic in the European Middle Ages

How many of you can name a medieval logician?

- “Late/High Middle Ages”: c1100–c1400.
- Inheritance of Aristotle and Boethius.
- Rise of the universities.
- Unique developments.

Why study it?

- Emphasis on "applied" logic: Language-based, multi-agent, dynamic.
- Different goals means different focuses, some of which provide new fodder for current research
- A case study: Temporal and spatial connectives.
  - Lambert of Lagny (13th C)
  - Roger Bacon (13th C)
  - Walter Burley (14th C)
  - William of Ockham (14th C)
  - Jean Buridan (14th C)

New binary logical connectives

- Modern propositional logic: Conjunctions, disjunctions, conditionals.
- 13th-14th C logicians: These three,

New binary logical connectives

- Modern propositional logic: Conjunctions, disjunctions, conditionals.
- 13th-14th C logicians: These three, plus causal, local, and temporal propositions.

Temporal propositions in the 13th century

Lambert of Auxerre, *Logica, sive Summa Lamberti*, mid 13th C:

> *A temporal proposition is one whose parts are joined by the adverb 'while'.*

Roger Bacon, *Art and Science of Logic*, mid 13th C:

> *local and temporal propositions differ from the other type of compound propositions because they are complex 'in virtue of a relation' rather than a connective. .*

Example: 'Socrates hauls the boat in when Plato runs.'

13th C truth conditions for temporal propositions

Anonymous, *Ars Burana*, c1200:

> *If the antecedent is false and the consequent true, the proposition is worthless (nugatoria).*

Lambert:

> *A temporal proposition is true if the two actions stated in the temporal proposition are carried out [at|in] the same time (in eodem tempore); it is false otherwise.*

Temporal propositions in the 14th century

- Ockham, Buridan, and Burley: Same syntactical definition as Lambert, with proviso that embedded temporal propositions are not allowed.
- Ockham and Burley: Extend analysis to other temporal adverbs, not only *dum* 'while, as long as, until', *quando* 'when, at which time', but also *ante* 'before', *post* and *postquam* 'after', and *priusquam* 'before, until'.

14th C truth conditions for temporal propositions

Walter Burley, *De Puritate Artes Logica*, early 14th C:

> *For the truth of a temporal [proposition], in which categorical propositions are conjoined by means of an adverb conveying simultaneity of time, it is required that both parts be true for the same time.*
> *For if the parts of such a temporal [proposition] are propositions of the present, then it is required that both parts be now true for this present time, and if it is of the past, it is required that both parts were true for some past time, this is, because they themselves were true in the present tense for some past time. And if they are propositions of the future, then it is required that both parts be true for some future time, that is, because they themselves will be true in the present tense for some future time.*

Inferences involving ‘while’ (1)

Corollary (Burley)

A temporal [proposition] implies both of its parts, and not conversely.

Corollary (Burley)

A temporal [proposition] implies a conjunction made of the temporal parts, but not conversely.

Corollary (Burley)

The negation (oppositum) of a temporal [proposition] is a disjunction composed from the opposites of those which were required for the truth of the temporal.

Note: This is a sufficient condition for falsity, not a necessary one.

Inferences involving 'while' (2)

William of Ockham, *Summa Logicae* Part II:

> *A conjunctive proposition follows from a temporal proposition—but not conversely. For this does not follow: 'Adam existed and Noah existed, therefore Adam existed when Noah existed'. Nor does this follow: 'Jacob existed and Esau existed, therefore Jacob existed when Esau existed'.*

Burley:

> *"Adam was when Noah was, therefore Adam was and Noah was" follows, but "Adam was and Noah was, therefore Adam was when Noah was" does not.*

The logic of while

Some medieval authors argue that temporal propositions are reducible to conjunctions, others that they are a type of conditional.

This is grounded in the intuition that there are two ways in which 'the same time' can be construed, existentially and universally:

1. $w \vDash pQq$ iff there exists w', $w' \vDash p$ and $w' \vDash q$.
2. $w \vDash pQq$ iff for all w', if $w' \vDash q$ then $w' \vDash p$.

(We use Q because of the Latin word *quando* 'while, at every time'.)

The two cases

Case (i) can be divided into three possibilities:

a $w = w'$

b $w < w'$,

c $w' < w$

The two cases

Case (i) can be divided into three possibilities:

a $w = w'$: when p and q are both present-tensed statements referring to now; pQq is equivalent to $p \wedge q$.

b $w < w'$,

c $w' < w$

The two cases

Case (i) can be divided into three possibilities:

a $w = w'$: when p and q are both present-tensed statements referring to now; pQq is equivalent to $p \wedge q$.

b $w < w'$, or

c $w' < w$: when point of reference is in past or in future, e.g., 'Socrates hauled the boat while Plato ran' or 'Socrates will haul the boat while Plato will run.'

The two cases

Case (i) can be divided into three possibilities:

a $w = w'$: when p and q are both present-tensed statements referring to now; pQq is equivalent to $p \wedge q$.

b $w < w'$, or

c $w' < w$: when point of reference is in past or in future, e.g., 'Socrates hauled the boat while Plato ran' or 'Socrates will haul the boat while Plato will run.'

Given that 'Socrates hauled the boat' and 'Plato ran' are past-tensed, it would be natural to formalize this as a 'while' connective between two past-tensed statements, e.g., $PpQPq$, in analogy to when the atomic statements are present-tensed.

The two cases

Case (i) can be divided into three possibilities:

a $w = w'$: when p and q are both present-tensed statements referring to now; pQq is equivalent to $p \wedge q$.

b $w < w'$, or

c $w' < w$: when point of reference is in past or in future, e.g., 'Socrates hauled the boat while Plato ran' or 'Socrates will haul the boat while Plato will run.'

Given that 'Socrates hauled the boat' and 'Plato ran' are past-tensed, it would be natural to formalize this as a 'while' connective between two past-tensed statements, e.g., $PpQPq$, in analogy to when the atomic statements are present-tensed. Hold that thought.

Past-tense while statements

But this doesn't get the truth conditions right.

- Ockham-Buridan-Burley: 'Socrates hauled the boat while Plato ran' being true implies that there was a time for which both propositions were true, i.e., $P(p \wedge q)$. This implies $Pp \wedge Pq$, but from $Pp \wedge Pq$, one cannot make the reverse inference.

Ockham says:

> *... a conjunctive proposition follows from a temporal proposition—but not conversely. For this does not follow: 'Adam existed and Noah existed, therefore Adam existed when Noah existed'. Nor does this follow: 'Jacob existed and Esau existed, therefore Jacob existed when Esau existed'.*

Whether $PpQPq$ implies $P(p \wedge q)$ depends on how, precisely, we interpret Q.

Two medieval questions

So:

1. Can 'while' be reduced to conjunction?
2. Can 'while' be reduced to implication?

Two medieval questions

So:

1. Can 'while' be reduced to conjunction?
2. Can 'while' be reduced to implication?

Well, what do modern logicians say?

'While' in modern temporal logic

- Usually found in the context of dynamic temporal logic, e.g., 'while φ, do α'.
- The non-imperative version is rare, with forward-looking $\mathcal{U}$ 'until' and the backward-looking $\mathcal{S}$ 'since' favored:

Definition (Weak until)

For $w \in W$:

$$w \vDash p\mathcal{U}q \quad \text{iff} \quad \begin{array}{l} \text{if there is a } w' \geq w \text{ s.t. } w' \vDash q \\ \text{then for every } w'',\ w \leq w'' < w', w'' \vDash p \end{array}$$

Malachi & Owicki's 'while'

Weak 'while' defined using weak 'until':

Definition (Malachi & Owicki 'while')

For $w \in W$:

$$
\begin{array}{lll}
w \vDash pQq & \text{iff} & w \vDash p\mathcal{U}(\neg q) \\
 & \text{iff} & \text{if there is a } w' \geq w \text{ s.t. } w' \vDash \neg q \\
 & & \text{then for every } w'',\ w \leq w'' < w', w'' \vDash p
\end{array}
$$

Problems with Malachi & Owicki's 'while'

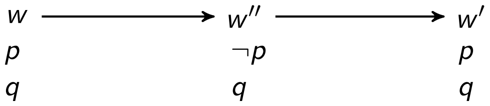

Figure: $w \vDash pQq$

- "p is true while q is true" defined to be: "p is true until q is false".
- But this English 'until' is not M&O's weak 'until', on which if q is always true, then p can be either true or false.

Manna & Pnueli's 'while'

Definition (Manna & Pnueli 'while')

For $w \in W$:

$$w \vDash pQq \quad \text{iff} \quad \begin{array}{l} w' \vDash p \text{ for every } w' \geq w \text{ such that} \\ w'' \vDash q \text{ for all } w'', w \leq w'' \leq w' \end{array}$$

(For every $w' \geq w$, if w'''s being between w and w' implies that $w'' \vDash q$, then $w' \vDash p$.)

Problems with Manna & Pnueli's 'while'

- When p and q are both present-tensed, if q is always false, pQq will always be true.
- When q is always false, w'''s being between w and w' does not imply that $w'' \vDash q$, and hence the antecedent of the conditional is falsified, making the entire condition satisfied.
- But this goes against the medieval requirement that pQq imply $p \wedge q$.

How to deal with past-tensed ‘while’ statements?

How to deal with past-tensed ‘while’ statements?

t_1	$\longrightarrow$	t_2	$\longrightarrow$	t_3	$\longrightarrow$	t_4	$\longrightarrow$	t_5
p		$\neg p$		Pp		Pp		Pp
$\neg q$		q		Pq		Pq		Pq

Figure: $t_3 \vDash PpQPq \wedge \neg P(p \wedge q)$

- $PpQPq$ does not (generally) imply $P(p \wedge q)$: When p and q are past-tensed statements, it is possible for them to both be true at the same time without there being any time for which the present-tense conjunction is true (see Figure), contra Ockham and Burley.
- On these conditions: ‘Socrates hauled the boat while Plato ran’ could not be formalized as a temporal compound of two past-tensed sentences.

The 'true-at' vs. 'true-for' distinction

- Lambert: the two actions are carried out "at the same time".
- Buridan, et al.: the two propositions are true "for the same time":

  It does not suffice for its categoricals to be true at the same time; for the propositions 'Aristotle existed' and 'The Antichrist will exist' are true at the same time, namely now, but it is required and sufficient that the copulas of the categoricals consignify the same time and that they be true for the same time, although not at that time.

In the Figure, Pp and Pq are *true at* the same time, namely t_3, but they are not *true for* the same time; Pp is true for t_1, while Pq is true for t_2.

Medieval 'while'

Definition (Medieval 'while')

For $w \in W$:

$$w \vDash pQq \quad \text{iff} \quad w \vDash p \wedge q \text{ and for all } w' \geq w$$
$$\text{if for all } w'', w \leq w'' < w', w'' \vDash q \text{ then } w' \vDash p$$

This analysis contains both conjunctive and implicative conditions.

‘Where’ propositions in the 13th Century

Almost entirely symmetrical to ‘while’ propositions:

Definition (Lambert)

A *local* proposition is one whose parts are joined by the adverb ‘where’, as in ‘Socrates hauls the boat where Plato runs.’

Definition (Bacon)

A *local* proposition is true if the two actions stated in the local proposition are carried out in the same place; it is false otherwise.

‘Where’ propositions in the 14th Century (1)

- Discussions are more detailed than 13th-C, but still circumscribed compared to temporal ones.
- Buridan’s definition of local propositions mirrors his definition of temporal ones, with the difference that ‘temporal adverb’ is replaced with ‘local adverb’ and ‘when’ with ‘where’ (though he notes it is possible to replace the local adverb with an equivalent phrase, such as ‘Socrates is in the place in which Plato is’.

Definition (Buridan)

A local proposition is true if “both categoricals [are] true for the place designated by the word ‘where’, and it is not sufficient that they are true for the same place”.

Thus, local propositions will entail conjunctions but not vice versa, as with temporal propositions.

'Where' propositions in the 14th Century (2)

Ockham allows local propositions to be composed of more than two categoricals, and notes:

> *[A local proposition] differs from a temporal one. For in order for a temporal proposition to be true it is required that both parts be true for the same place or for different places, while in order for a local proposition to be true it is required that both parts be true for the same place and not for different place.*

Not *entirely* symmetrical to temporal ones

1. truth of 'p where q' does not depend on the place of evaluation, whereas the truth of pQq depends on the time of evaluation: If no explicit time is specified, a temporal proposition must have both of its temporal parts true now; but if no explicit place is specified, a local proposition can be true without either of its parts being true here.
2. Space is extended in 3D, not linear.

Modern spatial logics: Logic of elsewhere

Due to von Wright, Segerberg: $\Box$ 'everywhere else'; $\Diamond$ 'somewhere else'; xRy iff $x \neq y$.

The logic of elsewhere is the smallest normal modal logic containing all instances of the schemata:

A: $p \rightarrow (\Box p \rightarrow \Box\Box p)$

B: $p \rightarrow \Box\Diamond p$

Modern spatial logics: Logic of elsewhere

Due to von Wright, Segerberg: $\Box$ 'everywhere else'; $\Diamond$ 'somewhere else'; xRy iff $x \neq y$.

The logic of elsewhere is the smallest normal modal logic containing all instances of the schemata:

A: $p \rightarrow (\Box p \rightarrow \Box\Box p)$

B: $p \rightarrow \Box\Diamond p$

This is the opposite of what we want!

Other modern spatial logics

Logics characterizing topological notions such as 'nearness' and 'distance' (Aiello & van Benthem; van Benthem & Bezhanishvili).

- Aimed at capturing "the ontological structure [of space]: What are the primitive objects and their relations?" [Aiello & van Benthem, p. 320]
- Not intended to look at "some existing human practice, e.g., a language with spatial expressions (say locative propositions) or a diagrammatic way of visualizing things" [ibid.].

Lambert's definition of 'where'

Lambert, quoting the anonymous author of the 12th-century *Book of Six Principles*, who took the definition from Aristotle's *Physics*:

> *"[W]here is the circumscription of a body proceeding from the circumscription of a place." For example, water collected in a container adopts the figure of the container and is transfigured in accord with the figure of the interior surface of the container, and the configuration that it has from the interior surface of the container is named 'where'.*

Regions and neighborhoods

Definition (Neighborhood)

A set $A \subset \mathbb{R}^2$ is a *neighborhood* of a point $(x, y) \in \mathbb{R}^2$ if A contains an open set containing (x, y).

Definition (Region)

Let $(x, y) \in \mathbb{R}^2$. We say that $\mathcal{R}(x, y) = A$ is a *region* of (x, y) if A is a simply connected neighborhood of (x, y).

Medieval 'where'

Definition (Medieval 'where')

For $(t, x, y) \in (\mathbb{R}^3, \leq)$:

$$
\begin{array}{lll}
(t, x, y) \vDash pUq & \text{iff} & \text{there is } x', y' \text{ s.t. } (t, x', y') \vDash p \wedge q \\
 & & \text{and for all } \mathcal{R}(x', y'), \\
 & & \text{if for all } (x'', y'') \in \mathcal{R}(x', y'), (t, x'', y'') \vDash q \\
 & & \text{then for all } (x'', y'') \in \mathcal{R}(x', y'), (t, x'', y'') \vDash p
\end{array}
$$

(We use U because of Latin *ubi* 'where, in what place'.)

Some concluding remarks

- An interesting medieval highlighting of the symmetry of temporal and spatial dimensions.
- We gave semantics for two new interesting binary operators.
- Further work: Axioms!

Thank you

and thanks to Nicholas Adams, Thomas Ball, Hadley Foster Barth, Melissa Barton, Kate Bell, Malin Berglund, Wendel Bordelon, Edward Boreham, Liam Kofi Bright, Edward Buckner, Don Campbell, Karen Carlisle, Erin Childs, Riia M. Chmielowski, Kay Ellis, Katherine Gensler, Andrew Grosser, Robyn Hodgkin, Justine Jacot, Esther Johnston, Earl P. Jones, Heather Rose Jones, Susanne Kalejaiye, Linse Rose Kelbe, Marleen de Kramer, Jennifer Knox, Barteld Kooi, Jean Kveberg, Christer Romson Lande, Lee Large, Dan Long, Christy Mackenzie, Dave Majors, Alex Malpass, Jennifer McGowan, Lesley McIntee, Liz McKinnell, Tom McKinnell, Sonia Murphy, Katherine Napolitano, Gabriela Aslı Rino Nesin, Paddy Neumann, Lynette Nusbacher, Peryn Westerhof Nyman, Caroline Orr, Sy Delta Parker, Susanne de Paulis, Judith Marie Phillips, Mike Prendergast, Daria Rakowski, Stephanie Rebours-Smith, Kevin Rhodes, Sarah Rossiter, Angela Sanders, Fiona Scerri, Amy Selman, Phil Selman, Jennifer Smith, Lena Thane-Clarke, Petra Träm, Joel Uckelman, Nicole Uhl, Rineke Verbrugge, Miesje de Vogel, Elmar Vogt, Ursula Whitcher, Brooke White, Nik Whitehead, and Anna Wilson.

Reference:

Uckelman, Sara L., "The Logic of Where and While in the 13th and 14th Centuries", in Lev Beklemishev, Stéphane Demri, & András Máté, eds., *Advances in Modal Logic* vol. 11 (College Publications, 2016): 535–550.