The Logic of While in the 13th and 14th Centuries

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Temporal propositions in the 13th century

Lambert of Auxerre, *Logica, sive Summa Lamberti*, mid 13th C:

> *A temporal proposition is one whose parts are joined by the adverb 'while', as in 'Socrates runs while Plato argues'.*

Roger Bacon, *Art and Science of Logic*, mid 13th C:

> *local and temporal propositions differ from the other type of compound propositions because they are complex 'in virtue of a relation' rather than a connective. Example: 'Socrates hauls [the boat] in when Plato runs'.*

13th C truth conditions for temporal propositions

Anonymous, *Ars Burana*, c1200:

> *If the antecedent is false and the consequent true, the proposition is worthless (nugatoria).*

Lambert:

> *A temporal proposition is true if the two actions stated in the temporal proposition are carried out [at/in] the same time (in eodem tempore); it is false otherwise.*

Temporal propositions in the 14th century

- Ockham, Buridan, and Burley: Same syntactical definition as Lambert, with proviso that embedded temporal propositions are not allowed.
- Ockham and Burley: Extend analysis to other temporal adverbs, not only *dum* 'while, as long as, until', *quando* 'when, at which time', but also *ante* 'before', *post* and *postquam* 'after', and *priusquam* 'before, until'.

14th C truth conditions for temporal propositions

Walter Burley, *De Puritate Artes Logica*, early 14th C:

> *For the truth of a temporal [proposition], in which categorical propositions are conjoined by means of an adverb conveying simultaneity of time, it is required that both parts be true for the same time.*
> *For if the parts of such a temporal [proposition] are propositions of the present, then it is required that both parts be now true for this present time, and if it is of the past, it is required that both parts were true for some past time, this is, because they themselves were true in the present tense for some past time. And if they are propositions of the future, then it is required that both parts be true for some future time, that is, because they themselves will be true in the present tense for some future time.*

Inferences involving ‘while’

Corollary

A temporal [proposition] implies both of its parts, and not conversely [Burley].

Corollary

A temporal [proposition] implies a conjunction made of the temporal parts, but not conversely [Burley].

Corollary

The negation (oppositum) of a temporal [proposition] is a disjunction composed from the opposites of those which were required for the truth of the temporal [Burley].

Note: This is a sufficient condition for falsity, not a necessary one.

'While' in modern temporal logic

- Usually in the context of dynamic temporal logic, e.g., 'while ϕ, do α'.
- The non-imperative version is rare, with forward-looking $\mathcal{U}$ 'until' and the backward-looking $\mathcal{S}$ 'since' favored:

Definition (Weak until)

For $w \in W$:

$$w \vDash p\mathcal{U}q \quad \text{iff} \quad \begin{array}{l}\text{if there is a } w' \geq w \text{ s.t. } w' \vDash q \\ \text{then for every } w'',\ w \leq w'' < w', w'' \vDash p\end{array}$$

Malachi & Owicki's 'while'

Weak 'while' defined using weak 'until':

Definition (Malachi & Owicki 'while')

For $w \in W$:

$$
\begin{array}{lll}
w \vDash pQq & \text{iff} & w \vDash p\mathcal{U}(\neg q) \\
 & \text{iff} & \text{if there is a } w' \geq w \text{ s.t. } w' \vDash \neg q \\
 & & \text{then for every } w'', \ w \leq w'' < w', w'' \vDash p
\end{array}
$$

Problems with Malachi & Owicki's 'while'

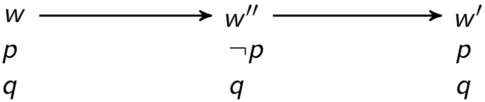

Figure: $w \vDash pQq$

- "p is true while q is true" defined to be: "p is true until q is false".
- English 'until' is not this weak 'until': if q is always true, then p can be either true or false.

Manna & Pnueli's 'while'

Definition (Manna & Pnueli 'while')

For $w \in W$:

$$w \vDash pQq \quad \text{iff} \quad \begin{array}{l} w' \vDash p \text{ for every } w' \geq w \text{ such that} \\ w'' \vDash q \text{ for all } w'', w \leq w'' \leq w' \end{array}$$

(For every $w' \geq w$, if w'''s being between w and w' implies that $w'' \vDash q$, then $w' \vDash p$.)

Problems with Manna & Pnueli's 'while' (1)

- When p and q are both present-tensed, if q is always false, pQq will always be true.
- When q is always false, w'''s being between w and w' does not imply that $w'' \vDash q$, and hence the antecedent of the conditional is falsified, making the entire condition satisfied.
- But this goes against the medieval requirement that pQq imply $p \wedge q$.

Problems with Manna & Pnueli's 'while' (2)

$$
\begin{array}{ccccccccc}
t_1 & \longrightarrow & t_2 & \longrightarrow & t_3 & \longrightarrow & t_4 & \longrightarrow & t_5 \\
p & & \neg p & & Pp & & Pp & & Pp \\
\neg q & & q & & Pq & & Pq & & Pq
\end{array}
$$

Figure: $t_3 \vDash PpQPq \wedge \neg P(p \wedge q)$

- $PpQPq$ does not imply $P(p \wedge q)$: When p and q are past-tensed statements, it is possible for them to both be true at the same time without there being any time for which the present-tense conjunction is true (see Figure 2), contra Ockham and Burley.
- On these conditions: 'Socrates lectured while Plato disputed' could not be formalized as a temporal compound of two past-tensed sentences.

The ‘true-at’ vs. ‘true-for’ distinction

- Lambert: the two actions are carried out “at the same time”.
- Buridan, et al.: the two propositions are true “for the same time”:

  > *It does not suffice for its categoricals to be true at the same time; for the propositions ‘Aristotle existed’ and ‘The Antichrist will exist’ are true at the same time, namely now, but it is required and sufficient that the copulas of the categoricals consignify the same time and that they be true for the same time, although not at that time.*

In Figure 2, Pp and Pq are *true at* the same time, namely t_3, but they are not *true for* the same time; Pp is true for t_1, while Pq is true for t_2.

Medieval ‘while’

Definition (Medieval ‘while’)

For $w \in W$:

$$w \vDash pQq \quad \text{iff} \quad w \vDash p \wedge q \text{ and for all } w' \geq w$$
$$\text{if for all } w'', w \leq w'' < w', w'' \vDash q \text{ then } w' \vDash p$$

An advantage of this account is that it helps understand why some medieval authors try to reduce temporal propositions to conjunctions and others to implications, because the truth conditions have both conjunctive and implicative conditions.

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