

# The Logic of (Where and) While in the 13th and 14th Centuries

Sara L. Uckelman  
s.l.uckelman@durham.ac.uk  
@SaraLUckelman

Advances in Modal Logic  
01 Sep 2016

## Temporal propositions in the 13th century

Lambert of Auxerre, *Logica, sive Summa Lamberti*, mid 13th C:

*A temporal proposition is one whose parts are joined by the adverb 'while', as in 'Socrates runs while Plato argues'.*

Roger Bacon, *Art and Science of Logic*, mid 13th C:

*local and temporal propositions differ from the other type of compound propositions because they are complex 'in virtue of a relation' rather than a connective. Example: 'Socrates hauls [the boat] in when Plato runs'.*

## 13th C truth conditions for temporal propositions

Anonymous, *Ars Burana*, c1200:

*If the antecedent is false and the consequent true, the proposition is worthless (nugatoria).*

Lambert:

*A temporal proposition is true if the two actions stated in the temporal proposition are carried out [at/in] the same time (in eodem tempore); it is false otherwise.*

## Temporal propositions in the 14th century

- Ockham, Buridan, and Burley: Same syntactical definition as Lambert, with proviso that embedded temporal propositions are not allowed.
- Ockham and Burley: Extend analysis to other temporal adverbs, not only *dum* 'while, as long as, until', *quando* 'when, at which time', but also *ante* 'before', *post* and *postquam* 'after', and *priusquam* 'before, until'.

## 14th C truth conditions for temporal propositions

Walter Burley, *De Puritate Artes Logica*, early 14th C:

*For the truth of a temporal [proposition], in which categorical propositions are conjoined by means of an adverb conveying simultaneity of time, it is required that both parts be true for the same time.*

*For if the parts of such a temporal [proposition] are propositions of the present, then it is required that both parts be now true for this present time, and if it is of the past, it is required that both parts were true for some past time, this is, because they themselves were true in the present tense for some past time.*

*And if they are propositions of the future, then it is required that both parts be true for some future time, that is, because they themselves will be true in the present tense for some future time.*

## Inferences involving 'while' (1)

### Corollary

*A temporal [proposition] implies both of its parts, and not conversely [Burley].*

### Corollary

*A temporal [proposition] implies a conjunction made of the temporal parts, but not conversely [Burley].*

### Corollary

*The negation (oppositum) of a temporal [proposition] is a disjunction composed from the opposites of those which were required for the truth of the temporal [Burley].*

Note: This is a sufficient condition for falsity, not a necessary one.

## Inferences involving 'while' (2)

*A conjunctive proposition follows from a temporal proposition—but not conversely. For this does not follow: 'Adam existed and Noah existed, therefore Adam existed when Noah existed'. Nor does this follow: 'Jacob existed and Esau existed, therefore Jacob existed when Esau existed' [Ockham].*

*"Adam was when Noah was, therefore Adam was and Noah was" follows, but "Adam was and Noah was, therefore Adam was when Noah was" does not [Burley].*

## Two medieval questions

- ① Can 'while' be reduced to conjunction?
- ② Can 'while' be reduced to implication?

## 'While' in modern temporal logic

- Usually in the context of dynamic temporal logic, e.g., 'while  $\phi$ , do  $\alpha$ '.
- The non-imperative version is rare, with forward-looking  $\mathcal{U}$  'until' and the backward-looking  $\mathcal{S}$  'since' favored:

### Definition (Weak until)

For  $w \in W$ :

$w \models p\mathcal{U}q$  iff if there is a  $w' \geq w$  s.t.  $w' \models q$   
then for every  $w''$ ,  $w \leq w'' < w'$ ,  $w'' \models p$

# Malachi & Owicki's 'while'

Weak 'while' defined using weak 'until':

## Definition (Malachi & Owicki 'while')

For  $w \in W$ :

$$\begin{aligned} w \models pQq & \text{ iff } w \models p\mathcal{U}(\neg q) \\ & \text{ iff } \text{if there is a } w' \geq w \text{ s.t. } w' \models \neg q \\ & \text{ then for every } w'', w \leq w'' < w', w'' \models p \end{aligned}$$

## Problems with Malachi & Owicki's 'while'

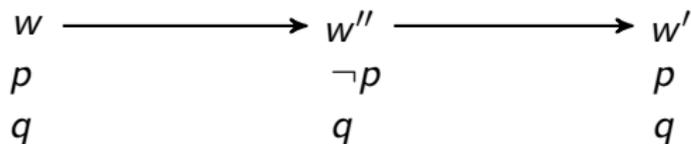


Figure:  $w \models pQq$

- “ $p$  is true while  $q$  is true” defined to be: “ $p$  is true until  $q$  is false”.
- But this English ‘until’ is not M&O’s weak ‘until’, on which if  $q$  is always true, then  $p$  can be either true or false.

# Manna & Pnueli's 'while'

## Definition (Manna & Pnueli 'while')

For  $w \in W$ :

$$w \models pQq \quad \text{iff} \quad w' \models p \text{ for every } w' \geq w \text{ such that} \\ w'' \models q \text{ for all } w'', w \leq w'' \leq w'$$

(For every  $w' \geq w$ , if  $w''$ 's being between  $w$  and  $w'$  implies that  $w'' \models q$ , then  $w' \models p$ .)

## Problems with Manna & Pnueli's 'while'

- When  $p$  and  $q$  are both present-tensed, if  $q$  is always false,  $pQq$  will always be true.
- When  $q$  is always false,  $w''$ 's being between  $w$  and  $w'$  does not imply that  $w'' \models q$ , and hence the antecedent of the conditional is falsified, making the entire condition satisfied.
- But this goes against the medieval requirement that  $pQq$  imply  $p \wedge q$ .

# How to deal with past-tensed 'while' statements?

## How to deal with past-tensed 'while' statements?

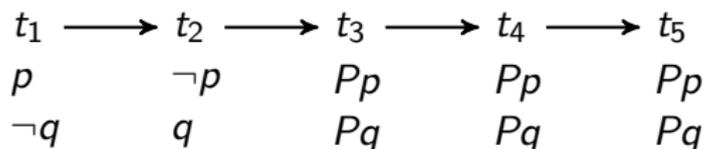


Figure:  $t_3 \models PpQPq \wedge \neg P(p \wedge q)$

- $PpQPq$  does not imply  $P(p \wedge q)$ : When  $p$  and  $q$  are past-tensed statements, it is possible for them to both be true at the same time without there being any time for which the present-tense conjunction is true (see Figure 2), contra Ockham and Burley.
- On these conditions: 'Socrates lectured while Plato disputed' could not be formalized as a temporal compound of two past-tensed sentences.

## The 'true-at' vs. 'true-for' distinction

- Lambert: the two actions are carried out “at the same time”.
- Buridan, et al.: the two propositions are true “for the same time”:

*It does not suffice for its categoricals to be true at the same time; for the propositions 'Aristotle existed' and 'The Antichrist will exist' are true at the same time, namely now, but it is required and sufficient that the copulas of the categoricals consignify the same time and that they be true for the same time, although not at that time.*

In Figure 2,  $Pp$  and  $Pq$  are true at the same time, namely  $t_3$ , but they are not true for the same time;  $Pp$  is true for  $t_1$ , while  $Pq$  is true for  $t_2$ .

# Medieval 'while'

## Definition (Medieval 'while')

For  $w \in W$ :

$w \models pQq$  iff  $w \models p \wedge q$  and for all  $w' \geq w$   
if for all  $w'', w \leq w'' < w', w'' \models q$  then  $w' \models p$

This analysis contains both conjunctive and implicative conditions.

## 'Where' propositions (1)

Almost entirely symmetrical to 'while' propositions:

### Definition (Lambert)

A *local* proposition is one whose parts are joined by the adverb 'where', as in 'Socrates runs where Plato argues'.

### Definition (Bacon)

A local proposition is true if the two actions stated in the local proposition are carried out in the same place; it is false otherwise.

## 'Where' propositions (2)

But ...

- truth of ' $p$  where  $q$ ' does not depend on the place of evaluation, whereas the truth of  $pQq$  depends on the time of evaluation: If no explicit time is specified, a temporal proposition must have both of its temporal parts true now; but if no explicit place is specified, a local proposition can be true without either of its parts being true here.
- Space is extended in 3D, not linear.

### Definition (Medieval 'where')

For  $(t, x, y) \in (\mathbb{R}^3, \leq)$ :

$(t, x, y) \models pUq$  iff there is  $x', y'$  s.t.  $(t, x', y') \models p \wedge q$   
and for all  $\mathcal{R}(x', y')$ ,  
if for all  $(x'', y'') \in \mathcal{R}(x', y')$ ,  $(t, x'', y'') \models q$   
then for all  $(x'', y'') \in \mathcal{R}(x', y')$ ,  $(t, x'', y'') \models p$

# Thank you

and thanks to Nicholas Adams, Thomas Ball, Hadley Foster Barth, Melissa Barton, Kate Bell, Malin Berglund, Wendel Bordelon, Edward Boreham, Liam Kofi Bright, Edward Buckner, Don Campbell, Karen Carlisle, Erin Childs, Riia M. Chmielowski, Kay Ellis, Katherine Gensler, Andrew Grosser, Robyn Hodgkin, Justine Jacot, Esther Johnston, Earl P. Jones, Heather Rose Jones, Susanne Kalejaiye, Linse Rose Kelbe, Marleen de Kramer, Jennifer Knox, Barteld Kooi, Jean Kveberg, Christer Romson Lande, Lee Large, Dan Long, Christy Mackenzie, Dave Majors, Alex Malpass, Jennifer McGowan, Lesley McIntee, Liz McKinnell, Tom McKinnell, Sonia Murphy, Katherine Napolitano, Gabriela Aslı Rino Nesin, Paddy Neumann, Lynette Nusbacher, Peryn Westerhof Nyman, Caroline Orr, Sy Delta Parker, Susanne de Paulis, Judith Marie Phillips, Mike Prendergast, Daria Rakowski, Stephanie Rebours-Smith, Kevin Rhodes, Sarah Rossiter, Angela Sanders, Fiona Scerri, Amy Selman, Phil Selman, Jennifer Smith, Lena Thane-Clarke, Petra Träm, Joel Uckelman, Nicole Uhl, Rineke Verbrugge, Miesje de Vogel, Elmar Vogt, Ursula Witcher, Brooke White, Nik Whitehead, and Anna Wilson.