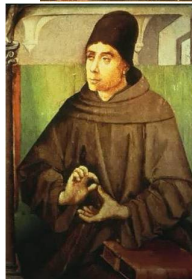
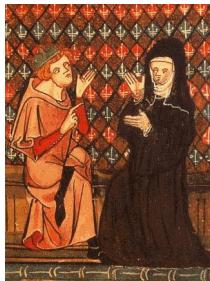


# Reasons Why You, Not a Medievalist, Should Be Interested in Medieval Logic

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@SaraLUckelman

Joint Mathematics and Philosophy Logic Seminar  
30 October 2019

# Why should modern logicians care about the history of logic?

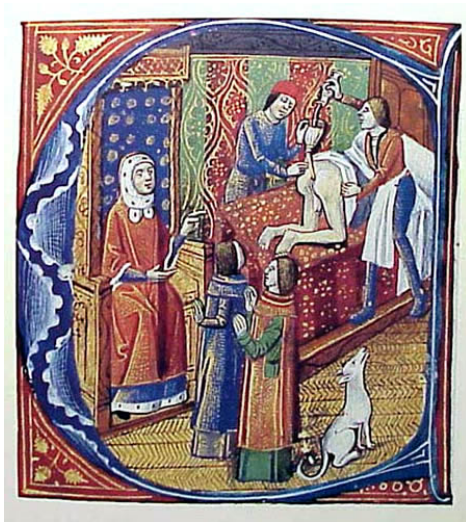


# There's something about logic...

... which is not like

- Medicine
- Biology
- Chemistry
- Astronomy

# History of Medicine



(source uncertain)

# History of Medicine



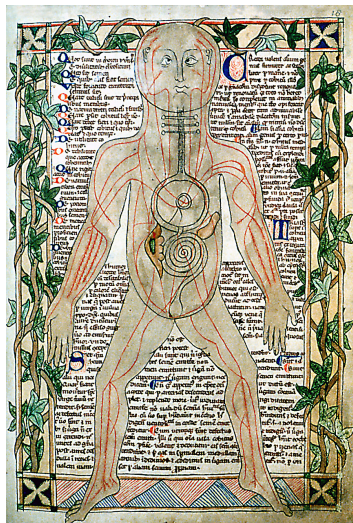
*Chirurgia*, Roger Frugard of Parma (c.1300–25)

# History of Medicine



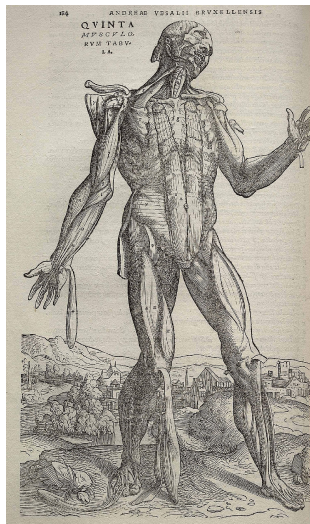
*Canon medicinae*, Avicenna (3q13thC)

# History of Biology



Medical Miscellany, Anonymous (c1292)

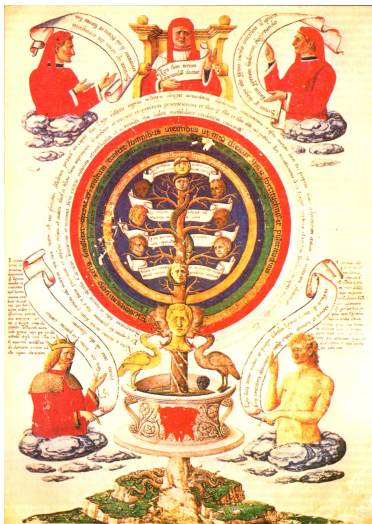
# History of Biology



*De humani corporis fabrica libri septem, Andreas Vesalius (1543)*



# History of Chemistry



Ramon Llull (16th C)

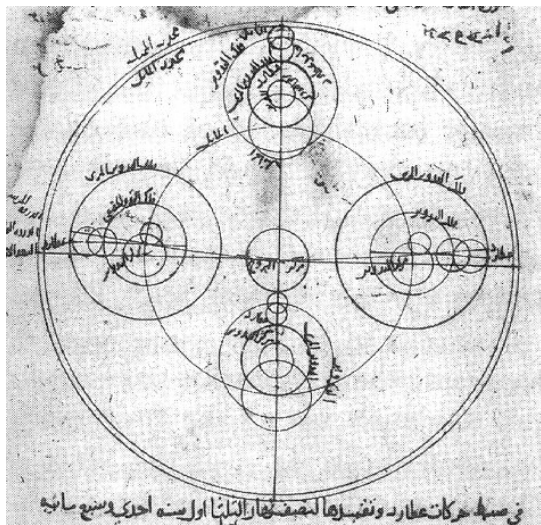
# History of Chemistry



Quelle: Deutsche Fotothek

*Konjunktion in der Kabbala, Stephan Michelspacher (1654)*

# History of Astronomy



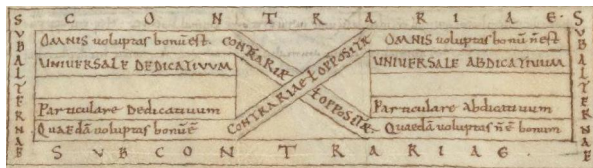
Ibn al-Shatir (14th C)

# Why is History of Logic different?



Apuleius, *Commentary on Aristotle's Perihermeneias*, (9th C)

# Why is History of Logic different?



Apuleius, *Commentary on Aristotle's Perihierarchias*, (9th C)

In many other sciences, lots of what we used to “know” is false.

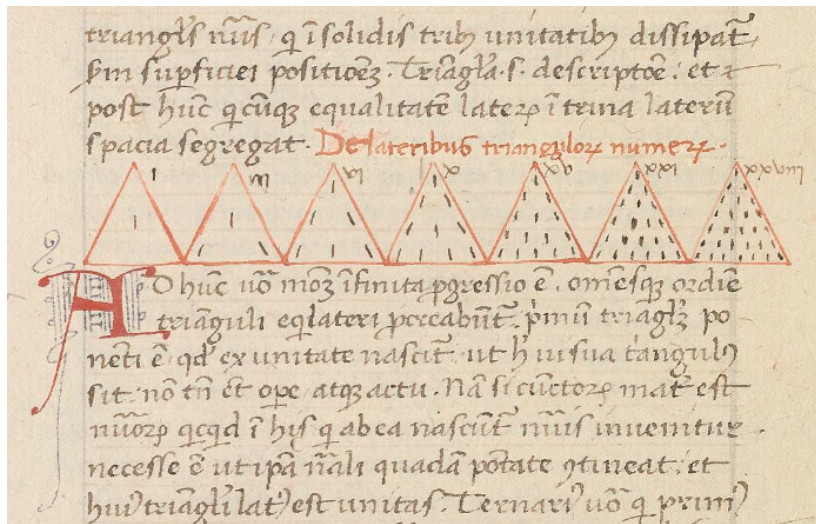
# History of Mathematics

[81] Another subalternate species of *subjection* is *subsuperpartient number*. It is the number contained in another plus its own two parts, or three, or four, or however many in another. Its species are *subsuperbipartient*, *subsupertripartient*, *subsuperquadripartient*, and so on. A *subsuperbipartient number* is one that is contained in another plus its own two-thirds or two-fifths parts, etc.; a *subsupertripartient number* is one that is contained in another plus its own three-fourths or three-fifths parts, etc., and so on, always comparing one species of *subsuperbipartient number* to one species of *superpartient number*.<sup>142</sup> [1.2.187-188]

[82] Another species of *subjection* is *submultiple subsuperparticular number*, whose subalternate species is *subdouble subsuperparticular*. Its species are: *subdouble subsesquialter*, *subdouble subsesquitertius*, *subdouble subsesquiquartus*, and so on.<sup>143</sup> Another subalternate species is *subtriple subsuperparticular*. The third species is *subquadruple subsuperparticular*, and so on indefinitely, whose lowest species are multiplied as stated in the first species, namely, on the basis of the division of *superparticular number*. [1.2.189-191]

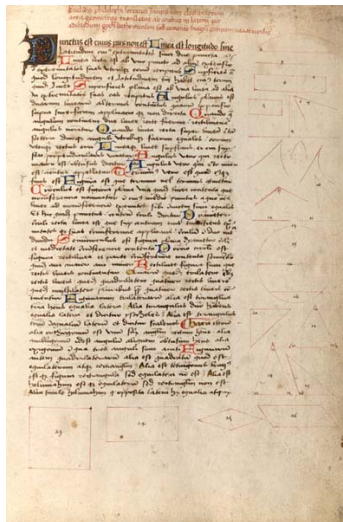
Roger Bacon, *The Art and Science of Logic*

# History of Mathematics



Boethius, *De institutione arithmetica* (15th C)

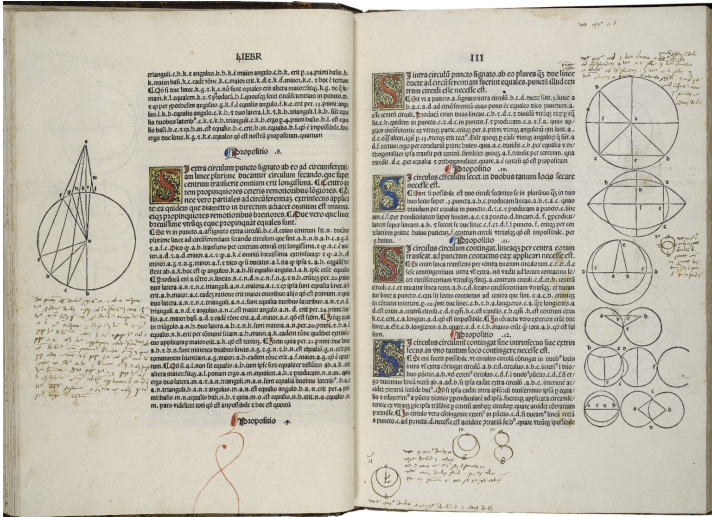
# History of Mathematics



Euclid, *Elements*, Sp Coll MS Gen. 1115 (France, c1480)



History of Mathematics



Euclid, Elements printed by Erhard Ratdolt (1482)

## Clarity is important!

“the greatest advance in logic since Aristotle”

[Green, Rossberg, & Ebert, 2015, p. 15]

$$\left[ \begin{array}{l} \vdash \varepsilon \left[ \begin{array}{l} \overbrace{\quad n \quad m \quad} \varepsilon \wedge (n \wedge \perp q) \\ m \wedge (\varepsilon \wedge \perp q) \\ n \wedge (n \wedge \perp q) \\ A = m; n \\ Iq \end{array} \right] \end{array} \right] = A \underline{\wedge} q$$

Frege, *Begriffsschrift*, vol. I, §158.

## De syllogismis

174

qualitas pter albedinem sed deberet sumi in minori albi  
 et for. et concludi igitur album est plato. Item non sequitur.  
 omnis suba est colorata papa est substantia igitur papa  
 est colorata: sed bene sequitur igitur papa est coloratus vel  
 coloratum. Sed bene sequitur omnis homo est albus: hec  
 mulier est homo igitur hec mulier est homo albus: sed ma-  
 ior est impossibilis: quia equialet isti impossibili omnis ho-  
 mo est homo albus.

**Quartus** modus prime figure. v.3. serio constat  
 implicite vel explicite ex vniuersali ne-  
 gatiua et particulari indefinita vel singulari affirmatiua: par-  
 ticularem indefinita singularem negatiua simplicite vel ex-  
 plicite directe aut indirecte concludentibus. verbi gratia. Et il-  
 lis premissis nullum animal est asinus et aliquid risibile est animal  
 sequitur quod aliquid risibile non est asinus. et est indirecte conclu-  
 dentibus quod aliquis asinus non est risibilis. Est tamen aduer-  
 tendum in syllogismis ampliatis quatuor concludit indire-  
 cte unde non sequitur. nullum animal erit albus for. est vel erit  
 album animal igitur aliquid album non erit for. et pono quod for. sit  
 omne album et quod nunquam post instans prius erit aliquid album  
 sed posse nigrum. Isto posito patet premissas esse veras si-  
 ne conclusionem: quia suum oppositum est verum. scilicet omne album  
 erit for. ut patet per exponentes: non igitur debet inferri talis  
 conclusio. quia si album in maiori fecit ampliatio sola pro fu-  
 turis et in conclusione indifferenter pro presenti et futuro id con-  
 cladi debet quod futurum album non erit for. quod verum est  
 qualitercunque sumatur.

Quintus modus prime figure. scilicet singularis constat

rum nullius quantitas est nec implicite nec explicite: quod tamen  
 requiritur ad syllogismum tam prime quam alterius figure. Et si de  
 sub maiore habet debitam proportionabile in predicato mi-  
 noris igitur est syllogismus in prima figura. negat prima. et si illud re-  
 quirat non tamen illud sufficit sed plura alia. Forte arguitur  
 omnia dicta in ista figura allegando Aristoteli. et Petrum hispa-  
 num ponentium nonne modos quatuor: concludentes directe  
 et quatuor indirecte. sed hoc non ponit nisi sex modi concludentes in-  
 differenter tam indirecte quam directe et est igitur positio insuf-  
 iciens. Et si hinc ut pluries responsum est quod ipsi talia non  
 posuerunt tanquam firma et vera: sed solum propter adfectus  
 ut citius caperent modum syllogismi. Et ex dictis in hac  
 prima figura sequuntur aliqua correlaria et regulariter. primum  
 est quod prima figura concludit omne genus problematis affirma-  
 tiuum et negatiuum vbi particulari indefinitum et singulare:  
 patet inspicenti modos. Secundum est quod in quatuor modis pri-  
 me figure minori existente negatiua nihil sequitur: aliter ex vero  
 concluditur falsum. ut omnis homo est suba nulla lapis est homo  
 igitur nullus lapis est suba. Et si de quibus sequitur omnis homo  
 est animal nullus asinus est homo igitur nullus asinus est animal. verum  
 est. sed hoc non est virtute syllogismi: sed quia prius est per se  
 necessarium. Unde non sequitur omnis homo est animal nullus asinus  
 est homo igitur nullus asinus est animal. Tertium est quod  
 in eisdem modis maiori existente particulari vel indefinita ni-  
 hil sequitur ut aliquid risibile est for. omnis homo est risibile igitur  
 omnis homo est for. Et si de quibus sequitur aliquid risibile est  
 animal omnis homo est risibilis igitur omnis homo est animal. Dicitur  
 tamen quod verum est non virtute syllogismi sed virtute huiusmodi con-

Paul of Venice, *Logica Magna*, (1499)

# Why does it matter?

*Those who cannot remember the past are condemned to repeat it.*



John Lydgate, *Troy Book and Siege of Thebes*, (BL MS Royal 18 D. ii, f. 30v., England, c1457)

# Why does it matter?

## DeMorgan's Law

$$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$$

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## DeMorgan's Law?

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*It should also be noted that the contradictory opposite of a conjunctive proposition is a disjunctive proposition composed of the contradictories of the parts of the conjunctive.*

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[Ockham, Summa Logicae II, chs. 32, 33]

# How is history of logic different?

- General approach to modalities
- The Liar and other paradoxes
- Temporal and spatial logics
- Dynamic and multi-agent logics
- Lying and deceit
- Knowledge and uncertainty
- The role of grammar in reference

# How is history of logic different?

- General approach to modalities \*
- The Liar and other paradoxes
- Temporal and spatial logics \*
- Dynamic and multi-agent logics
- Lying and deceit
- Knowledge and uncertainty \*
- The role of grammar in reference



# General approach to modalities



*We commonly use the verb 'to do' in place of all other verbs, regardless of the signification of these other verbs and regardless of whether they are finite or infinite. In fact, 'to do' may even stand for 'not to do'. If you think about it carefully, you will see that when we ask about someone 'What (how) is he doing?' here 'doing' stands for any verb that can be given in answer. And so too, these other verbs stand for the verb "to do". For in a correct reply to one who asks "What (how) is he doing?" any verb at all will indicate a doing on the part of the person asked about. If someone were to respond, "He is reading" or "He is writing", it is the same as if he were saying, "He is doing this, namely, reading", or "He is doing that, namely, writing" [Anselm of Canterbury, Philosophical Fragments]*

# Temporal and spatial logics

Prior (obviously).

# Temporal and spatial logics

Prior (obviously).

But also: ' $p$  while  $q$ ' and ' $p$  where  $q$ ':

*A temporal proposition is true if the two actions stated in the temporal proposition are carried out at the same time; it is false otherwise.*

*A local proposition is true if the two actions stated in the local proposition are carried out in the same place; it is false otherwise*  
*[Lambert of Auxerre, Summa Lamberti]*

# Temporal and spatial logics

*For if the parts of such a temporal [proposition] are propositions of the present, then it is required that both parts be now true for this present time, and if it is of the past, it is required that both parts were true for some past time, this is, because they themselves were true in the present tense for some past time. And if they are propositions of the future, then it is required that both parts be true for some future time, that is, because they themselves will be true in the present tense for some future time [Burley, De Puritate Artis Logicae]*

# Temporal and spatial logics

## Definition (Malachi & Owicki 'while')

For  $w \in W$ :

$$\begin{aligned} w \models pQq \quad \text{iff} \quad & w \models p\mathcal{U}(\neg q) \\ \text{iff} \quad & \text{if there is a } w' \geq w \text{ s.t. } w' \models \neg q \\ & \text{then for every } w'', w \leq w'' < w', w'' \models p \end{aligned}$$

## Definition (Manna & Pnueli 'while')

For  $w \in W$ :

$$w \models pQq \quad \text{iff} \quad \begin{aligned} & w' \models p \text{ for every } w' \geq w \text{ such that} \\ & w'' \models q \text{ for all } w'', w \leq w'' \leq w' \end{aligned}$$

# Temporal and spatial logics

## Definition (Medieval 'while')

For  $w \in W$ :

$$w \models pQq \quad \text{iff} \quad \begin{array}{l} w \models p \wedge q \text{ and for all } w' \geq w \\ \text{if for all } w'', w \leq w'' < w', w'' \models q \text{ then } w' \models p \end{array}$$

# Knowledge and uncertainty

*Every proposition which someone considers and which he does not know to be true nor know to be false is doubtful to him.  
[William Heytesbury, Regula Solvendi Sophismata]*

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$$U\phi \leftrightarrow \neg K\phi \wedge \neg K\neg\phi$$



## Knowledge and uncertainty

Consider the case where “you firmly and unwaveringly believe, as you do in fact, that Antichrist will come; and I suppose further that no Antichrist will come”.

- you are certain about the proposition ‘Antichrist will come’
- you do not know that it is true (because it is false)
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*To doubt is to consider a proposition but, because of various reasons for or against it, neither to believe firmly that it is true nor to believe firmly that it is false; thus every proposition to which someone gives sufficient consideration, and which he understands but neither believes to be true nor believes to be false, is doubtful to that person [Paul of Venice, Logica Magna]*

But *why* is it different?

- Logic as timeless truth?
- Changing conception of logic?

You've convinced me, now what?

Not everyone is going to go out and learn Latin and medieval palaeography.

# You've convinced me, now what?

Not everyone is going to go out and learn Latin and medieval palaeography.

- *Cambridge Companion to Medieval Logic*, Stephen Read & Catarina Dutilh Novaes, eds., pub. Oct. 2016.
- A 13th-century reading list:
  - ▶ Roger Bacon, *The Art and Science of Logic*, trans. T.S. Maloney.
  - ▶ Lambert of Auxerre, *Logica or Summa Lamberti*, trans. T.S. Maloney.
  - ▶ Peter of Spain, *Summaries of Logic*, trans. B. Copenhaver, T. Parsons.
  - ▶ William of Sherwood, *Introduction to Logic and Syncategorematic Terms*, trans. N. Kretzmann.
- Social media:  
<https://www.facebook.com/groups/medievallogic/>,  
<https://medievallogic.wordpress.com/>.