

# What Problem Did Ladd-Franklin (Think She) Solve(d)?

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# The Solution

## Theorem

*The argument of inconsistency,*

$$(a \nabla b)(\bar{b} \nabla c)(c \vee a) \nabla \quad (\text{II})$$

*is the single form to which all the ninety-six valid syllogisms (both universal and particular) may be reduced [Ladd, 1883, p. 40].*

## Proof.

Any given syllogism is immediately reduced to this form by taking the contradictory of the conclusion, and by seeing that the universal propositions are expressed with a negative copula and particular propositions with an affirmative copula [Ladd, 1883, p. 40]. □

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## Where the present talk started from

- Not only did I have no idea what this solution was,
- I also had no idea what problem it solved.

What is the “form” of a syllogism?

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- Is it “figure”?

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- Is it “figure”?
- Is it “mood”?



# Syllogisms: A primer (1)

## Definition

A *categorical proposition* is a subject-predicate proposition consisting of a subject term, a predicate term, a quality, and a quantity.

There are four types of categorical proposition recognised by Aristotle:

- a “All  $S$  are  $P$ ” (universal affirmative)
- e “No  $S$  is  $P$ ” (universal negative)
- i “Some  $S$  is  $P$ ” (particular/partial affirmative)
- o “Some  $S$  is not  $P$ ” (particular/partial negative)

## Syllogisms: A primer (2)

### Definition

A *syllogism* is a set of three categorical propositions which share amongst them three terms that each occur exactly twice. Two of the propositions are designated the premises, and the other is the conclusion.

The predicate term of the conclusion is the 'major' term; the subject term of the conclusion is the 'minor' term; the term that occurs only in the premises is the "middle term". It is a convention that the premise with the major term in it, the major premise, is written first.

## Syllogisms: A primer (2)

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The predicate term of the conclusion is the ‘major’ term; the subject term of the conclusion is the ‘minor’ term; the term that occurs only in the premises is the “middle term”. It is a convention that the premise with the major term in it, the major premise, is written first.

**Note:** This is *Aristotelian* rather than *Aristotle*.

## Syllogisms: A primer (3)

	Ist Figure	IIInd Figure
Major premise:	$P - M$	$M - P$
Minor premise:	$M - S :$	$M - S :$
Conclusion:	$P - S$	$P - S$

  

	IIIrd Figure	IVth Figure
Major premise:	$P - M$	$M - P$
Minor premise:	$S - M :$	$S - M :$
Conclusion:	$P - S$	$P - S$

Figure: The Four Figures

# Syllogisms: A primer (4)

Ist figure	$P a M$	,	$M a S$	:	$P a S$	<b>Barbara</b>
	$P e M$	,	$M a S$	:	$P e S$	<b>Celarent</b>
	$P a M$	,	$M i S$	:	$P i S$	<b>Darii</b>
	$P e M$	,	$M i S$	:	$P o S$	<b>Ferio</b>
	$P a M$	,	$M a S$	:	$P i S$	<b>Barbari</b>
	$P e M$	,	$M a S$	:	$P o S$	<b>Celaront</b>
IIInd figure	$M e P$	,	$M a S$	:	$P e S$	<b>Cesare</b>
	$M a P$	,	$M e S$	:	$P e S$	<b>Camestres</b>
	$M e P$	,	$M i S$	:	$P o S$	<b>Festino</b>
	$M a P$	,	$M o S$	:	$P o S$	<b>Baroco</b>
	$M e P$	,	$M a S$	:	$P o S$	<b>Cesaro</b>
	$M a P$	,	$M e S$	:	$P o S$	<b>Camestrop</b>
IIIrd figure	$P a M$	,	$S a M$	:	$P i S$	<b>Darapti</b>
	$P i M$	,	$S a M$	:	$P i S$	<b>Disamis</b>
	$P a M$	,	$S i M$	:	$P i S$	<b>Datisi</b>
	$P e M$	,	$S a M$	:	$P o S$	<b>Felapton</b>
	$P o M$	,	$S a M$	:	$P o S$	<b>Bocardo</b>
	$P e M$	,	$S i M$	:	$P o S$	<b>Ferison</b>
IVth figure	$M a P$	,	$S a M$	:	$P i S$	<b>Bramantip</b>
	$M a P$	,	$S e M$	:	$P e S$	<b>Camenes</b>
	$M i P$	,	$S a M$	:	$P i S$	<b>Dimaris</b>
	$M e P$	,	$S a M$	:	$P o S$	<b>Fesapo</b>
	$M e P$	,	$S i M$	:	$P o S$	<b>Fresison</b>
	$M a P$	,	$S e M$	:	$P o S$	<b>Camenop</b>

## Reprise: What is the “form” of a syllogism?

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## Reprise: What is the “form” of a syllogism?

- Is it “figure”?  
No.
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## Reprise: What is the “form” of a syllogism?

- Is it “figure”?

No.

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No.



## Reprise: What is the “form” of a syllogism?

- Is it “figure”?

No.

- Is it “mood”?

No.

- Is it something else?

Hardly likely.

## Comments on Ladd's achievement

*In 1883, while a student of C.S. Peirce at Johns Hopkins University, Christine Ladd-Franklin published a paper titled On the Algebra of Logic, in which she develops an elegant and powerful test for the validity of syllogisms that constitutes the most significant advance in syllogistic logic in two thousand years. . . In this paper, I bring to light the important work of Ladd-Franklin so that she is justly credited **with having solved a problem over two millennia old** [Russinoff, 1999, p. 451, emphasis added].*

*The result was the **ground-breaking** discovery involving the reduction of Aristotelian syllogistics into a single formula [Pietarinen, 2013, p. 3, emphasis added] [preprint]; The result was the reduction of the Aristotelian syllogistics into a single formula" [Pietarinen, 2013, p. 142] [published].*

# The Five Algebras of Logic

Ladd identifies five algebras of logic, due to:

- Boole (in *Laws of Thought*)
- Jevons [Jevons, 1864]
- Schröder [Schröder, 1877]
- McColl [McColl, 1877]
- Peirce [Peirce, 1867]

Her aim was to introduce a sixth that addresses what she sees as are the drawbacks of the previous attempts.

# Terms

The basic component of Ladd's algebras is subject and predicate terms. Atomic subject and predicate terms (hereafter simply called 'terms') are indicated by, e.g.,  $a$ ,  $b$ ,  $c$ . Ladd follows Wundt and Peirce in using  $\infty$  as a term to represent the domain of discourse [Ladd, 1883, p. 19].

Complex terms can be formed from atomic terms as follows:

- $\bar{a}$  = "what is not  $a$ ".
- $a \times b$  = "what is both  $a$  and  $b$ ".
- $a + b$  = "What is either  $a$  or  $b$ ".

At times,  $a \times b$  will be represented as  $ab$ . Infinite series of  $\times$  or  $+$ , or combinations of the two, are allowed.  $\overline{\infty}$  is given its own symbol:  $0$  [Ladd, 1883, p. 19].

# What can be done with terms

- The identity of the subject and predicate terms can be affirmed.
- The identity of the subject and predicate terms can be denied.
- Complex terms can be negated (Ladd identifies three ways: Boole/Jevons; DeMorgan; Schröder).

## Identity propositions

There are two types of propositions in Ladd's algebra: "those which affirm the identity of the subject and predicate, and those which do not"  
[Ladd, 1883, p. 17].

In all six of the algebras under consideration (the original five plus Ladd's), identity propositions are expressed the same way, via equality:

$$a = b \tag{1}$$

Note that while Ladd uses  $a$  and  $b$  here, these identity propositions are not restricted to atomic terms; complex terms may also be used in place of  $a$  and  $b$ .

## Some true identity propositions involving only positive terms

$$\begin{array}{l|l} aaa = a & a + a + a = a \\ abc = bca = cba & a + b + c = b + c + a = c + b + a \\ a(b + c) = (ab + ac) & a + bc = (a + b)(a + c) \end{array}$$

That is,  $\times$  and  $+$  are both idempotent, associative, and commutative, and  $\times$  and  $+$  both distribute over each other.

## How identity should be understood

- NOT as the identification of two objects, picked out via constants in the logical language.
- BUT as indicating the intersubstitutability of the two logical expressions  $a$  and  $b$ , *salve veritate*.

$a = b$  is equivalent to the following two propositions [Ladd, 1883, p. 18]:

There is no  $a$  which is not  $b$ . (2)

and

There is no  $b$  which is not  $a$ . (3)



# Categorical propositions in algebra (1)

We can either

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We can either

*assign the expression of the 'quantity' of propositions to the copula or to the subject [Ladd, 1883, p. 23].*

- If quantity is assigned to the copula, then two copulas are necessary (one universal, one partial).
- If it is assigned to the subject term, the only copula that is needed is identity.

## Categorical propositions in algebra (2)

	Traditional	Boole / Schröder	Jevons	McCull	Peirce
Uni- versal	All $a$ is $b$	$a = vb$	$a = ab$	$a : b$	$a < b$
	No $a$ is $b$	$a = v\bar{b}$	$a = a\bar{b}$	$a : \bar{b}$	$a < \bar{b}$
Part- ial	Some $a$ is $b$	$va = vb$	$ca = cab$	$a \div \bar{b}$	$a \bar{<} \bar{b}$
	Some $a$ is not $b$	$va = v\bar{b}$	$ca = ca\bar{b}$	$a \div b$	$a \bar{<} b$

- First type: McCull and Peirce
- Second type: Boole, Schröder, Jevons. In Boole and Schröder's version, the symbol  $v$  should not be taken as a categorical term like  $a$  or  $b$ , but rather a special term that picks out some arbitrary indefinite class. Jevons's  $c$  works similarly, but he does not distinguish it in the way that  $v$  is distinguished; it can be any other class term [Ladd, 1883, p. 24].

## Which way is preferable?

Advantages of the second way:

- Only one copula is necessary.

Advantages of the first way:

- Copulas that include their quantity can be used to link either terms *or* propositions, so that, e.g.,  $a\bar{\supset}b$  can be read either “*a* is not wholly contained under *b*” or “*a* does not imply *b*” [Ladd, 1883, p. 24].
- There is a correspondence between the quantity of the copula and its quality. The universal copulas are positive (affirmative), and the partial copulas are negative [Ladd, 1883, p. 25].

## Ladd's way

Instead of taking as basic:

- (a)  $a \prec b$  “ $a$  is wholly  $b$ ”
- (o)  $a \overline{\prec} b$  “ $a$  is not wholly  $b$ ”

take:

- (e)  $a \overline{\vee} b$  “ $a$  is-wholly-not  $b$ ”
- (i)  $a \vee b$  “ $a$  is-partly  $b$ ”

## DeMorgan's eight propositions

$a \bar{\vee} b$  “ $a$  is-not  $b$ ; “no  $a$  is  $b$ ”.

$a \vee b$  “ $a$  is in part  $b$ ; “some  $a$  is  $b$ ”.

$a \bar{\vee} \bar{b}$  “ $a$  is-not not- $b$ ; “all  $a$  is  $b$ ”.

$a \vee \bar{b}$  “ $a$  is in part not- $b$ ; “some  $a$  is not- $b$ ”.

$\bar{a} \bar{\vee} b$  “what is not- $a$  is-not  $b$ ; “ $a$  includes all  $b$ ”.

$\bar{a} \vee b$  “what is not- $a$  is in part  $b$ ; “ $a$  does not include all  $b$ ”.

$\bar{a} \bar{\vee} \bar{b}$  “what is not- $a$  is-not not- $b$ ; “there is nothing besides  $a$  and  $b$ ”.

$\bar{a} \vee \bar{b}$  “what is not- $a$  is in part not- $b$ ; “there is something besides  $a$  and  $b$ ”.

# Symmetry

Both  $\vee$  and  $\bar{\vee}$  are **symmetric** combinators: “the propositions  $a \bar{\vee} b, a \vee b$ , may be read either forward or backward” [Ladd, 1883, p. 26]; inclusion statements using  $\prec$  are **asymmetric**.

Inclusions can be converted into exclusions by changing the copula and the sign of the predicate [Ladd, 1883, p. 27]:

$$a \prec b = a \bar{\vee} \bar{b}$$

Every exclusion is equivalent to a pair of inclusions, differing on which of the two terms you take as the predicate:

$$\begin{aligned} a \bar{\vee} b &= a \prec \bar{b} = b \prec \bar{a} \\ a \vee b &= a \bar{\prec} \bar{b} = b \bar{\prec} \bar{a} \end{aligned}$$

## Advantages of this approach

- Categoricals and hypotheticals are treated identically in the formal system [Ladd, 1883, p. 23].
- If we let  $p$  denote a premise and  $c$  a conclusion following from  $p$ , then we can express this consequent fact as either:

$$p \supset c$$

or

$$\bar{c} \supset \bar{p}$$



## Existence and nonexistence claims

We can express existence and nonexistence claims via  $\infty$ :

$$x \nabla \infty \tag{4}$$

means “x does not, under any circumstances, exist”, and

$$x \vee \infty \tag{5}$$

means that “x is at least sometimes existence” [Ladd, 1883, p. 29].

## Translating between propositions and terms

Unlike in the (pure) algebra of terms, the algebra of propositions does not have 0 [Ladd, 1883, p. 29]. As a result, we can drop reference to  $\infty$  in contexts where it can be restored without ambiguity. Therefore, we can write (4) and (5) as:

$$x\bar{\vee} \tag{4'}$$

and

$$x\vee \tag{5'}$$

This notation allows us to translate from categorical propositions (e.g.,  $a\bar{\vee}b$  “No  $a$  is  $b$ ”) into statements about terms (e.g.,  $ab\bar{\vee}$  “The combination  $ab$  does not exist”) [Ladd, 1883, p. 30].

# The three subjects of symbolic logic

Ladd identifies three subjects of interest for any symbolic logic [Ladd, 1883, p. 30]:

- the uniting and separating of propositions.
- the insertion or omission of terms, or immediate inference.
- elimination with the least possible loss of content, or syllogism.

## On the elimination of terms

*The most common object in reasoning is to eliminate a single term at a time—namely, one which occurs in both premises [Ladd, 1883, p. 37].*

This goal of logic can be accomplished via the inference form “if  $a$  is  $b$  and  $c$  is  $d$ , then  $ac$  is  $bd$  [Ladd, 1883, p. 34]:

$$(a \bar{\vee} b)(c \bar{\vee} d) \bar{\vee} (ac \vee b + d) \quad (I)$$

By setting  $d$  equal to  $\bar{b}$ , so that  $b + d = \infty$ , we can rewrite (I) as:

$$(a \bar{\vee} b)(\bar{b} \bar{\vee} c)(c \vee a) \bar{\vee} \quad (II)$$

(going via the intermediate equation  $(a \bar{\vee} b)(c \bar{\vee} \bar{b})(ac \vee \infty) \bar{\vee}$ ).

# The main result

## Theorem

*The argument of inconsistency,*

$$(a \nabla b)(\bar{b} \nabla c)(c \vee a) \nabla \quad (\text{II})$$

*is the single form to which all the ninety-six valid syllogisms (both universal and particular) may be reduced [Ladd, 1883, p. 40].*

## Proof.

Any given syllogism is immediately reduced to this form by taking the contradictory of the conclusion, and by seeing that the universal propositions are expressed with a negative copula and particular propositions with an affirmative copula [Ladd, 1883, p. 40]. □

## Rules for the validity of a syllogism

From this theorem a corollary follows, in the form of an easy to apply rule, stated in ordinary English, for identifying whether any syllogism is valid:

### Rule

Take the contradictory of the conclusion, and see that the universal propositions are expressed with a negative copula and particular propositions with an affirmative copula. If two of the propositions are universal and the other particular, and if that term only which is common to the two universal propositions has unlike signs, then, and only then, the syllogism is valid [Ladd, 1883, p. 41].

## Two examples (1)

### Example

The syllogism Baroco:

All  $P$  is  $M$   
Some  $S$  is not  $M$   
 $\therefore$  Some  $S$  is not  $P$

is equivalent to the inconsistency

$$(P \nabla \bar{M})(S \vee \bar{M})(S \nabla P) \nabla$$

## Two examples (2)

### Example

The syllogism Bocardo:

Some  $M$  is not  $P$   
All  $M$  is  $S$   
 $\therefore$  Some  $S$  is not  $P$

is equivalent to the inconsistency

$$(M \vee \bar{P})(M \nabla \bar{S})(S \nabla \bar{P})\nabla$$



## The solution was already in Aristotle

The rudiments of this is already incorporated in the Aristotelian reductions, (as was noticed by Kattsoff in 1936):

*This method is actually the method of indirect reduction which is denoted by the letter 'k' in the mnemonic names Baroko and Bokardo [sic]. The name antilogism was given to this by Mrs. Ladd-Franklin [Kattsoff, 1936, p. 385].*

# Aristotle already knew how to do this

Aristotle reduced both

<b>Baroco</b>	<b>Bocardo</b>
All $P$ are $M$	Some $M$ is not $P$
Some $S$ is not $M$	All $M$ are $S$
$\therefore$ Some $S$ is not $P$	$\therefore$ Some $S$ is not $P$

to

<b>Barbara</b>
All $M$ are $P$
All $S$ are $M$
$\therefore$ All $S$ are $P$

via *reductio ad absurdum*—that is, taking the contradictory of the conclusion and replacing one of the premises with it, and then making the contradictory of the replaced premise the conclusion.

## How what Ladd does differs

However, instead of taking Barbara as basic and Baroco and Bocardo as derived, Ladd showed that one can take as “basic” the inconsistency of the following three claims:

All  $M$  are  $P$

All  $S$  are  $M$

Not all  $S$  are  $P$

Any two of these propositions entails the denial of the third; which is to say that the contradictory of any of the propositions follows from the other two, which gives us all three syllogisms.

## What's the benefit of doing this? (1)

First, Ladd says:

*If for the usual three statements consisting of two premises and a conclusion one substitutes the equivalent three statements that are together incompatible. . . one has a formula which has this great advantage: the order of the statements is immaterial—the relation is a perfectly symmetrical one [Ladd-Franklin, 1928, p. 532].*

In addition to the symmetry of the relation, the result is

*a source of great simplicity—there is only one valid form of the antilogism instead of the fifteen valid forms of the syllogism which common logic requires us to bear in mind [Ladd, 1883, p. 532].*

## What's the benefit of doing this? (2)

Thirdly, both the simplicity and the symmetry can be improved upon if all of the three claims can be written as either (e) “No  $S$  are  $P$ ” or (i) “Some  $S$  is  $P$ ” claims, which can be simply converted.

If we admit ‘infinite’ terms, then this rewriting into symmetric propositions is always possible, as “All  $S$  are non- $P$ ” is equivalent to “No  $S$  is  $P$ ”, and “Some  $S$  is not  $P$ ” is equivalent to “Some  $S$  is non- $P$ ” (cf. [Reichenbach, 1952, p. 1]).

## What did Ladd think she was doing?

*The task which Boole accomplished was the complete solution of the problem:—given any number of statements, involving any number of terms mixed up indiscriminately in the subjects and the predicates, to eliminate certain of those terms, that is, to see exactly what the statements amount to irrespective of them, and then to manipulate the remaining statements so that they shall read as a description of a certain other chosen term (or terms) standing by itself in a subject or predicate [Franklin, 1889, p. 543].*

The simplest example of this type of manipulation is found in the syllogism—three propositions, six terms, each occurring twice, and one term which is eliminated from the premises when generating the conclusion. The difficulty in Boole's solution is determining what is the term (or terms) to be eliminated; once the term is identified, "an ordinary syllogism would suffice to put it to flight" [Franklin, 1889, p. 544].

## Russinoff thinks was Aristotle's problem

*The problem that Aristotle posed and attempted to solve is to give a general characterization of the valid syllogisms [Russinoff, 1999, p. 452].*

But she gives no evidence for this!

*... though he did not succeed in providing a unified and complete treatment of the syllogistic argument. . . [Russinoff, 1999, pp. 453–454]*

But why do we think he wanted to/intended to/thought he had to?

# What counts as a problem?

- Does a problem have to be recognised as a problem for it to be a problem?
- Did the 'problem' that Ladd solved exist through the two millennia in which no one was bothered by it?
- Or did it only become a problem once someone found it problematic?



## A bit of historiography (1)

In the earliest review of the paper, [Anonymous, 1883], no specific mention is made of this result. The (unidentified) reviewer introduces Ladd's new notation,  $\vee$  and  $\bar{\vee}$ , gives its semantics and formation rules, and notes that “with these she is able to write algebraically all the old forms of statement, and to perform the customary operations of symbolic logic with great brevity and facility” [Anonymous, 1883, p. 514].

The singling out of the antilogism as a fundamental contribution is first (as far as I can tell) made by Brown: “when Mrs. Ladd-Franklin has demonstrated that one simple form underlies all syllogism. . .” [Brown, 1909, p. 304].

## A bit of historiography (2)

Shen quotes the late Professor Josiah Royce of Harvard:

*There is no reason why this should not be accepted as the definitive solution of the problem of the reduction of syllogisms. It is rather remarkable that the crowning activity in a field worked over since the days of Aristotle should be the achievement of an American woman" [Shen, 1927, p. 60].*

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*Royce told his students, "It is rather remarkable that the crowning activity in a field worked over since the days of Aristotle should be the achievement of an American woman. "Professor Royce on an American Woman's Work," New York Evening Post, n.d., Box 14, CLF-FF Papers [Spillman, 2012, fn. 29].*

*In a newspaper clip "To Get Her Degree Earned Years Ago", Josiah Royce is quoted as describing her thesis work as "the crowning activity in a field worked over since the days of Aristotle". "The [Aristotelian] system was never fully demonstrated until Mrs. Ladd-Franklin worked out the whole method at Johns Hopkins" (The Hartford Courant, February 21, 1926, p. 20) [Pietarinen, 2013, fn. 6].*

## One last evaluation of the contribution

*See Russinoff (1999) on how, in her dissertation, Ladd-Franklin in fact managed to solve — or at least to see the solution to — the problem that was over two millennia old, though she did not give, nor could she have given the proof in such a rigorous form that is possible nowadays in the semantic terms of possible interpretations in varying domains” [Pietarinen, 2013, fn. 6].*

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Everyone assumes we know what “the problem” is!

## Concluding remarks

- In her 1883 dissertation, Ladd-Franklin introduced to Boolean algebra a pair of symmetric copula.
- This allowed her to define the “antilogism”, an “inconsistent triad” that could be used to represent every valid syllogism.
- People recognised the utility of this representation soon after her work.
- Within 30 years, people made the leap to her formula being a **solution** to a **problem**.
- Within 40 years, people attributed the problem to Aristotle.
- At some point after that, the problem attributed to Aristotle was attributed as a problem to all intervening logicians, too.
- While she might have solved a problem, it certainly wasn't Aristotle's, nor had it vexed people for millennia.

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