

# Everything You Wanted to Know About Medieval Logic But Were Too Embarrassed To Ask

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# The inheritance of Aristotle (1)

*Anicius Manlius Severinus Boëthius* (c475–524): “The last of the Roman philosophers, and the first of the scholastic theologians” (Martin Grabmann).

- theological treatises
- *Consolation of philosophy*
- commentaries on Aristotle
- *On hypothetical syllogisms*
- translations of Aristotle
  - ▶ *Categories*
  - ▶ *On interpretation*
  - ▶ *Prior analytics*
  - ▶ *Topics*
  - ▶ *Sophistical refutations*

## The inheritance of Aristotle (2)

Until 1100:

- Aristotle, *Categories*
- Aristotle, *On interpretation*
- Porphyry, *Introductions*.

c1120: Rediscovery of Boëthius' translations of

- Aristotle, *Prior Analytics*
- Aristotle, *Topics*
- Aristotle, *Sophistical Refutations*

c1150: James of Venice translates

- Aristotle, *Posterior Analytics*
- Aristotle, *On the soul*
- Aristotle, *Metaphysics*

# Dialectic in the 11th and 12th centuries (1)

Berengar of Tours (c1000–1088) vs. Lanfranc of Bec (c1005–1089)

- Analysis of pronouns in *Hoc corpus est meum*.
- Application of Aristotelian distinction between substance and accident.
- First use of purely dialectical techniques to arbitrate theological problems.

## Dialectic in the 11th and 12th centuries (3)

### Anselm of Canterbury (c1033–1109)

- Distinction between *usus loquendi* and *usus proprie* applied to theological discussions.
- Modal conception of agency
- Ontological argument

## Dialectic in the 11th and 12th centuries (3)

- Adam of Petit Pont
- William of Soissons
- John of Salisbury
- Peter Abelard
- Numerous anonymous texts

# The birth of the universities (1)

The earliest universities:

- **University of Bologna**, established c.1200. Non-Bolognese students form interest groups, the so-called *nationes* or *universitates*:
  - ▶ *universitas legistarum citramontanorum* (law students from this side of the mountains),
  - ▶ *universitas legistarum ultramontanorum* (law students from the other side of the mountains),
  - ▶ *universitas artistarum et medicorum* (arts and medicine students),
  - ▶ *collegium doctorum* (college of doctors, for the teachers).

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  - ▶ *collegium doctorum* (college of doctors, for the teachers).
- **University of Paris**, established c.1200. Parisian educational institutions planned a more systematic way of teaching organization, forming a *universitas*.
  - ▶ *Facultas Artium* (Faculty of arts; later called the Faculty of philosophy).
  - ▶ *Facultas Iurisprudentiae* (Faculty of law).
  - ▶ *Facultas Medicinae* (Faculty of medicine).
  - ▶ *Facultas Theologiae* (Faculty of theology).

# The birth of the universities (2)

- *Trivium (artes sermocinales)*
  - ▶ *Grammar*
  - ▶ *Rhetoric*
  - ▶ *Dialectic/Logic*
- *Quadrivium (Artes reales / Artes physicae)*
  - ▶ *Arithmetic*
  - ▶ *Geometry*
  - ▶ *Astronomy*
  - ▶ *Music*

# 13th-century developments

- Rediscovery of Aristotelian logic texts sparks new areas of research.
- The birth of the universities provides a framework for that new research.
- Embedding of Aristotelian teaching into Christian philosophy.

# The new logic (1)

The 13th and 14th centuries were the pinnacle of the development of the *logica nova*, both integrating traditional Aristotelian logic and going beyond it.

- Developments of Aristotelian logic:
  - ▶ Categorical syllogisms: Mnemonics, reductions, fourth figure.
  - ▶ Relational syllogisms:
  - ▶ Modal syllogisms (esp. influenced by Arabic commentators on Aristotle)
- Beyond Aristotle:
  - ▶ Properties of terms
    - ★ signification
    - ★ supposition
  - ▶ Fallacies
  - ▶ *Obligationes*
  - ▶ *Insolubilia* and *sophismata*

## Some prominent 13th-century logicians

- William of Sherwood (c1200–c1272).
- Nicholas of Paris (fl. 1250).
- Lambert of Auxerre/Laigny (fl. 1250).
- Peter of Spain/Petrus Hispanus (fl. 2h13thC).
- Roger Bacon (c1219–c1292).
- John Duns Scotus (c1266–1308).
- Simon of Faversham (fl. 1280s).

## Some prominent 14th-century logicians

- Walter Burley (c1275–1344).
- Pseudo-Scotus. Modalities: *dubium*, *scitum*, *opinatum*, *volitum*, *dilectum*.
- William of Ockham (c1295–1349). *Entia non sunt multiplicanda praeter necessitatem*.
- Thomas Bradwardine (c1295–1349). New solution to the Liar paradox.
- Jean Buridan (c1300–1361).
- William Heytesbury (c1310–1372). Oxford calculator.
- Roger Swyneshed (c1310–1365).
- Albert of Saxony (1316–1390).
- John Wyclif (1320s–1384).
- Marsilius of Inghen (1340–1396).
- Paul of Venice (1369–1429).
- Peter of Mantua (fl. 1390s).

## Properties of terms: *Syncategoremata*

Distinction of terms into **categoremata** and **syncategoremata**:

- **Categorematic terms**: terms which have signification (meaning) on their own.
  - ▶ dog, cat, Socrates, mammal, animal, chimera, goat-stag.
- **Syncategorematic terms**: terms which only signify in conjunction with categorematic terms.
  - ▶ Common syncategorematic terms: all, some, none, no, only, alone, except, unless, if, begins, ceases, necessarily, contingently, and, or...

Development of **terminist** logic (properties of terms) and theories of consequence (what follows from what).

## Properties of terms: *Significatio*

The signification of a term is conventional:

*Significatio est intellectus rei ad quem intellectum rei vox imponitur ad voluntatem instituentis [Lambert of Auxerre].*

Concepts pick out things and terms signify concepts and this signification comes about by a mental act of some person.

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Moving from **analysis of meaning of terms** (signification) to **analysis of meaning of terms in phrases**: Supposition of a term = what the term stands for in a sentence.

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- *homo est animal* personal supposition
- *homo est species* natural supposition
- *homo est disyllabum* material supposition

# Supposition (1)

What effect do quantifiers have?

- *homo est philosophus*
- *omnis homo est philosophus*
- *paene omnis homo est philosophus*

## Supposition (2)

Under what conditions is *omnis homo philosophus est* true?

- If *philosophus* supposits for every instance of *homo* (*suppositio mobilis*).
- Instantiation: *Aristoteles homo est. Aristoteles philosophus est.* But, *Aristophanes homo est et non est philosophus.*

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Under what conditions is *omnis homo praeter Socratem philosophus est* true?

- If *philosophus* supposits for every instance of *homo* except for Socrates.
- Instantiation: *Aristoteles homo est.*  
*Aristoteles praeter Socrates philosophus est.*  
*(suppositio immobilis).*

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*homo est disyllabum.*

*aliquis homo est disyllabum.*

*aliquis disyllabum est homo.* (simple conversion)

*disyllabum est homo.*

## Supposition (3)

### Consequences for logic:

- Whether conversion rules can be applied depends on the type of supposition in the proposition.
- Addition of ampliators such as past and future tense markers, modal operators, opaque contexts changes the supposition of terms.

In *Omnis homo est currens*, *homo* supposits for all currently existing men. In *Omnis homo erit currens*, *homo* supposits for all currently and futurely existing men.

# Theories of consequences

Articulating a general theory of “what follows from what”:

- Formal vs. material
- Conceptual containment/relevance
- Necessary truth preservation

# Theories of consequences

Articulating a general theory of “what follows from what”:

- Formal vs. material: When it is said that “an animal is a substance; therefore a man is a substance” is a good consequence I reply that this consequence does not hold in virtue of form (*ratione formae*), but rather in virtue of matter. Because according to the Commentator [Averroes] on the first book of the *Physics*, an argument which is valid (*concludens*) in virtue of form must hold in all matter. This consequence, however, holds only for features which are essential . . . and so this consequence is not formal (*formalis*).
- Conceptual containment/relevance
- Necessary truth preservation

# Theories of consequences

Articulating a general theory of “what follows from what”:

- Formal vs. material
- Conceptual containment/relevance: Therefore, inference consists in the necessity of consecution, that is, in that the sense (sententia) of the consequent is required (exigitur) by the sense (sensus) of the antecedent, as is asserted with a hypothetical proposition. . .
- Necessary truth preservation

# Theories of consequences

Articulating a general theory of “what follows from what”:

- Formal vs. material
- Conceptual containment/relevance
- Necessary truth preservation: Hence, many say that of two propositions one is antecedent to the other if it is impossible for the one to be true without the other being true, and one is consequent to the other if it is impossible for the one not to be true when the other is true, so that every proposition is antecedent to every other proposition for which it is impossible for it to be true without the other being true.

# Obligationes

**Obligationes**: A game-like disputation, whose name derives from the fact that one of the players is “obliged” to follow certain rules.

The origin of *obligationes* is unclear, as is their purpose.

Different types of *obligationes*:

- *positio* (most common)
- *depositio*
- *dubitatio*
- *impositio*
- *petitio*
- *rei veritas / sit verum*

## A brief introduction to *obligationes* (1)

- Two players: Opponent and Respondent.
- Opponent puts forwards propositions.
- Respondent must concede, deny, doubt in accordance with certain rules.
- Goal of Respondent: follow the rules and maintain consistency.

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## A brief introduction to *obligationes* (2)

- *positio, depositio, dubitatio*:
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- *positio, depositio, dubitatio*:
  - ▶ The Respondent's primary obligation is to accept / deny / be doubtful about the initial proposition.
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- *petitio, impositio*:
  - ▶ The Opponent asks the Respondent to reply in a certain way.
  - ▶ The Opponent changes the imposition of a word (or words).
- *sit verum / rei veritatis*:
  - ▶ “Let it be true that. . .” / “The truth of things is. . .”

## *Obligationes* according to Burley (1).

Burley defines the general goal of an *obligatio* as follows:

*The opponent's job is to use language in a way that makes the respondent concede impossible things that he need not concede because of the positum. The respondent's job, on the other hand, is to maintain the positum in such a way that any impossibility seems to follow not because of him but rather because of the positum.*

⇒ the goal is consistency, not logical truth or validity.

### References

- Burley, W., *Tractatus de obligationibus*, in R. Green, ed., *An introduction to the logical treatise 'de obligationibus'* 2 (1963).
- Burley, W., *Obligations (selections)*, in N. Kretzmann & E. Stump, eds., *The Cambridge translations of medieval philosophical texts 1: logic and the philosophy of language* (Cambridge University Press, 1988): 369–412.

## *Obligationes* according to Burley (2).

**General Rule 1** Everything following from an *obligatum* must be conceded (where '*obligatum*' is interpreted as what has been conceded or what must necessarily be conceded).

**General Rule 2** Everything incompatible with the *obligatum* must be denied.

**General Rule 3** One must reply to what is irrelevant in accordance with its own quality.

### Definition

A proposition is *irrelevant* or *impertinent* if neither it nor its negation follows from the set of propositions which have already been conceded (which includes the negations of propositions which have been denied).

## *Positio* according to Burley.

### Rule

Everything that is posited and put forward in the form of the positum during the time of the *positio* must be conceded.

### Rule

Everything that follows from the positum must be conceded. Everything that follows from the positum either together with an already conceded proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be conceded.

### Rule

Everything incompatible with the positum must be denied. Likewise, everything incompatible with the positum together with an already conceded proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be denied.

## An example *positio*.

Suppose  $\phi$  does not imply  $\neg\psi$  and  $\phi$  is known to be false.

	<b>Opponent</b>	<b>Respondent</b>
1	$\phi$ .	

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3	$\psi$	

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1	$\phi$ .	I admit it.
2	$\neg\phi \vee \psi$ .	I concede it.
3	$\psi$	I concede it.

### Note

This example shows how, given a *positum* which is false, but not inconsistent, the Opponent can force the Respondent to concede any other consistent proposition.

## A more interesting example

### Opponent

- 1  $\phi$  or  $\phi$  must be conceded.

### Respondent

I admit it.

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- 1  $\phi$  or  $\phi$  must be conceded.
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I admit it.  
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- 1  $\phi$  or  $\phi$  must be conceded.
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### Respondent

- I admit it.  
I deny it.  
I conceded it.

## A more interesting example

### Opponent

- 1  $\phi$  or  $\phi$  must be conceded.
- 2  $\phi$  must be conceded.
- 3  $\phi$  follows from the positum and the opposite of something correctly denied
- 4  $\phi$  must be conceded.

### Respondent

I admit it.  
I deny it.  
I conceded it.  
??

## *Dubitatio* according to Nicholas of Paris.

### Rules

- if  $\varphi$  or  $\neg\varphi$  is equivalent with the *dubitatum*,  $\varphi$  must be doubted.
- if  $\varphi$  implies the *dubitatum*, it must be doubted or denied.
- if  $\varphi$  is implied by the *dubitatum*, it must be doubted or accepted.
- if  $\varphi$  is irrelevant, the **respondent** should accept if he knows  $\varphi$  is true, deny if he knows  $\varphi$  is false, and doubt if he does not know either.
- the exercise cannot be terminated (!)
- world-knowledge does not change (“all responses must be directed to the same instant”).

## Another example *positio* (1)

From *Obligationes Parisienses*, an anonymous early 13th C MS [de Rijk 1975]

*For example: It is posited that Antichrist exists. You must put it as 'Antichrist is', this is the positum and the propositum under the same form of speech with which it was posed. Therefore it is conceded. . . Then it is put forward 'The Antichrist is colored'. This is something false. . . See therefore whether it may follow, putting it in the consequent of a conditional and the positum in the antecedent, in this way: 'If the Antichrist exists, the Antichrist is colored'. It is agreed that this is true. Therefore, the Antichrist being colored is a consequence of the positum. Therefore, it is conceded. You must put it in this way: 'Antichrist is white'. This is something false. Consider whether something may be previously conceded or previously negated, or nothing. It is agreed that something is previously conceded. See therefore whether it follows, by fashioning a conditional, namely that 'if the Antichrist [is] and the Antichrist is colored, the Antichrist is white'. This conditional is false. Therefore 'Antichrist being colored' is false and doesn't follow from the positum. Therefore it is refused. Therefore it may be negated. Then in this manner 'Antichrist is not neutral'. This is something true. And it is agreed that something is previously conceded and something previously negated. See therefore whether the Antichrist not being neutral is repugnant, putting down to the opposite of this in the consequent of a conditional, in this way 'if the Antichrist [is], and the Antichrist is colored, and the Antichrist is not white, the Antichrist is neutral'. This conditional is false. Therefore the Antichrist being neutral does not follow. Therefore the Antichrist being not neutral is not repugnant. And it is true. Therefore it is true and not repugnant. Therefore it is conceded. With what is conceded is put forth that 'Antichrist is black'. This is something false. It is agreed that somethings are conceded and something is negated. See therefore whether Antichrist being black is a consequence of the positum and the conceded things and the opposite of the negated thing, in this way: 'if the Antichrist is and the Antichrist is colored and the Antichrist is not white and the Antichrist is not neutral, the Antichrist is black'. This conditional is true. Therefore Antichrist being black is a false sequence. Therefore it is conceded.*

## Another example *positio* (2)

0.	Antichrist exists.	Admit	False, but consistent.
1.	Antichrist is.	Concede	Same as the <i>positum</i> .
2.	Antichrist is colored.	Concede	Follows from (1).
3.	Antichrist is white.	Deny	False, and does not follow from (1), (2).
4.	Antichrist is not neutral.	Concede	True, and not inconsistent with (1), (2), and the denial of (3).
5.	Antichrist is black.	Concede	False, but follows from (1), (2), the opposite of (3), (4).

## How *sit verum* differs from *positio* (1)

*And rei veritatis differs from positio because when rei veritatis has been done, then concerning any thing irrelevant or not following, it is not refused, but on the other hand if positio has been done, then it is to be refused. When it is said 'rei veritatis is that Antichrist exists', then concerning this: 'Antichrist is white' the response is 'prove it!', but if positio has been done the response to the same is: 'It is false!'" [de Rijk, 1975, p. 28].*

“Antichrist is white”

*Positio*

*Sit verum*

# “Antichrist is white”

## *Positio*

- 1 “I posit that Antichrist exists”. Admit.

## *Sit verum*

- 1 “Let it be true that Antichrist exists”. Admit.

# “Antichrist is white”

## *Positio*

- 1 “I posit that Antichrist exists”. Admit.
- 2 “Antichrist is colored”. Concede, as before.

## *Sit verum*

- 1 “Let it be true that Antichrist exists”. Admit.
- 2 “Antichrist is colored”. Concede, for same reason.

# “Antichrist is white”

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# “Antichrist is white”

## *Positio*

- 1 “I posit that Antichrist exists”. Admit.
- 2 “Antichrist is colored”. Concede, as before.
- 3 “Antichrist is white”. Deny:
  - ▶ “If Antichrist exists and is colored, then he is white” is false.
  - ▶ “If Antichrist exists and is colored, then he is not white” is false.
  - ▶ “Antichrist is white” is false because Antichrist does not in fact exist (and hence he cannot have a color).

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## *Sit verum*

- 1 “Let it be true that Antichrist exists”. Admit.
- 2 “Antichrist is colored”. Concede, for same reason.
- 3 “Antichrist is white”. Doubt, because it is assumed that Antichrist does exist, and thus he must be white or black, but it is not known which.

## The goal and purpose of *obligationes*

The goal and purpose of obligational reasoning is one of the big unsolved questions in medieval logic.

- academic exercises
- early forays into axiomatic systems
- for counterfactual reasoning
- for evaluating *sophismata* and *insolubilia*
- showing co-tenability of a set of propositions.
- theory of belief revision
- thought experiments
- forerunner of modern thesis disputations

## The goal and purpose of *obligationes*

King explains the apparent “content-freeness” of obligational disputations by pointing out that

*they operate at a higher level of logical generality than that at which substantive debate occurs. If this is correct, then actual obligational moves—perhaps even recognized as such—are the vehicle whereby real argument takes place*

and thus *obligationes* provide a “meta-methodology” for reasoning.

### Reference:

King, P. “Opposing and responding: comments on Paul Spade”, preprint, 2004,  
[http://individual.utoronto.ca/pking/presentations/Spade\\_Comments.pdf](http://individual.utoronto.ca/pking/presentations/Spade_Comments.pdf)

## Insolubles & *sophismata* (1)

Developed in the 12th century in both **grammatical** and **logical** contexts.

A **sophisma** is a sentence whose analysis either leads to an apparent contradiction, or for which two equally plausible analyses can be given, one for its truth and one for its falsity.

*Insolubilia. . . dicuntur a sophistis aliqua insolubilia non quia nullo modo possunt solvi, sed quia cum difficultate solvuntur [William of Ockham, III-3, 46].*

## Insolubles & *sophismata* (2)

The most famous insoluble: **the Liar**.

This sentence is false.

$\varphi$  :  $\varphi$  is false.

Five types of medieval solutions to the Liar can be identified:

- *secundum quid et simpliciter*.
- *transcasus*.
- Distinction between the exercised act and the signified act.
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  - ▶ Johannes Duns Scotus, *Questiones*.
  - ▶ The exercised act of the liar is “speaking the truth”.
  - ▶ The signified act of the liar is “speaking a falsehood”.
  - ▶ The liar expresses something which is not the truth, so it is false.

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- *cassatio*. **Solution.** The Liar sentence does not have a truth value.
  - ▶ If you are uttering an insoluble, you are saying nothing.
  - ▶ Therefore an insoluble has the same truth value as the empty utterance: none.

## Insolubles & *sophismata* (5)

The most productive era in the theory of insolubles was from 1320 to 1350.

- Thomas Bradwardine
- Roger Swyneshed
- William Heytesbury
- John Wyclif (c1330–1384).
- Peter of Ailly (*Petrus de Alliaco*; 1350–1420).

## Where to go from here?

- Stephen Read & Catarina Dutilh Novaes, eds., *Cambridge Companion to Medieval Logic*.
- Norman Kretzmann, et al., eds., *Cambridge History of Later Medieval Philosophy*.
- Medieval Logic & Semantics blog:  
<http://medievallogic.wordpress.com/>
- Medieval Logic Facebook group:  
<https://www.facebook.com/groups/medievallogic/>