

Formalization and Medieval Logic

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Introduction

- ① Medieval logic and formality
- ② Formalization examples
- ③ Limitations of formalizations
- ④ Why do we formalize?
- ⑤ How do we formalize?

Medieval logic and formality

- The Latin of medieval logicians: Formal, regimented.
- Regimented nature: common ground between medieval and modern logicians.
- Modern logicians have something to say about medieval logic in so far as the language and arguments of the latter are as formal as those used by the former.

Some examples

- ① Malink's reconstruction of Aristotle's modal syllogistic
- ② Johnston's reconstruction of Buridan's modal syllogistic
- ③ The additive (Dutilh Novaes) and reductive (Uckelman) models of *obligationes*
- ④ Ampliation as a form of quantified temporal logic (Uckelman)
- ⑤ Read's analysis of Bradwardine's theory of signification and solution to the Liar Paradox

Is there really common ground?

E.J. Ashworth ("Terminist Logic," 2010):

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Why?

The limits of formality

- **Formal validities** vs. **material validities**.
 - ▶ valid in virtue of the form of the argument, valid for all subject matter
 - ▶ valid in virtue of something other than the form of the argument, valid for only some subject matter, valid in virtue of the subject matter.

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- **Formal and material validities** vs. **“dialectical” validities**.
 - ▶ MacFarlane’s Formality Thesis (“There are inferences whose validity is not grounded in any external fact”)

Performative aspects of logic

- Logic/dialectic essentially involves agents who reason.
- Inference/reasoning is an action or a process.
- These processes always take place in a particular context.
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- Importance of the role of epistemology:
 - ▶ in the Middle Ages “logic... had a straightforwardly cognitive orientation” (Ashworth, 2010, p. 147).
 - ▶ “Medieval authors generally did not operate with the conception of logical omniscience that is included in some modern theories... They treat the logic of epistemic notions from the point of view of factual attitudes” (Knuuttila, *Modalities in Medieval Philosophy* 1993, p. 177).

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Failure to take into account these facts can lead to a miscategorization of medieval logic from the point of view of modern logic.

Why do we formalize

- To gain ideas that drive forward modern logic.
 - ① “where”/“while” operators
 - ② Liar paradox
 - ③ *dubitatio*
- To help understand/clarify what medieval authors are saying.
 - ▶ Model building
 - ▶ Definitions and rules
 - ▶ Formalizing proofs
 - ▶ Identifying gaps/holes

How do we formalize?

- Need to understand/identify purpose *first!*
- Our goal will direct the development of our formal tools

The importance of the definitions

- Getting the definitions right makes logic easy
- Getting the definitions right is hard!

An object lesson in definitions

- ① Are there statements which are neither true nor false?
- ② Are there statements which are both true and false?
- ③ Is there a difference between truth and necessary truth?
- ④ Is there a difference between falsity and necessary falsity?
- ⑤ Can we argue about or with statements whose truth value we don't know?
- ⑥ Is there one correct mode of reasoning, or does correct reasoning vary depending on context?

What does it mean to get the definitions “right”? (1)

- right definitions get you easy proofs
- right definitions allow you to prove true things

What is needed is **appropriate coordination between the definitions and the world**

What does it mean to get the definitions “right”? (2)

Getting definitions “right” requires making decisions about what is important:

- What parts of the world will you highlight?
- What parts of the world will you sideline?

Is this argument valid? (1)

All mammals are animals.
All cats are mammals.
Therefore all cats are animals.

Fix a definition of validity: “An argument is valid if every model of the premises is also a model of the conclusion.”

Is this argument valid? (2)

All mammals are animals.

All cats are mammals.

Therefore all cats are animals.

$A a M$	p	$\forall x(Mx \rightarrow Ax)$
$M a C$	q	$\forall x(Cx \rightarrow Mx)$
$A a C$	r	$\forall x(Cx \rightarrow Ax)$
Y	N	Y

What about this argument?

All mammals are animals.

All cats are mammals.

Therefore some cat is an animal.

$A a M$	p	$\forall x(Mx \rightarrow Ax)$
$M a C$	q	$\forall x(Cx \rightarrow Mx)$
$A i C$	r	$\exists x(Cx \wedge Ax)$
Y	N	N

Validity is not constant

How we define our language, how we define our models determines which translations are valid and which are not.

What does this tell us about our original argument?

Why definitions matter

- Getting your language right determines what is valid, what you can express, what you can prove.
- Decisions make evident what you think is relevant/important/valuable/worth tracking.

These decisions are not neutral!

An object lessons for my students

- Take-home mid-term exam
- ~ 1.5 pages of prose and one question to answer: “Should Captain Aenne Qadu d’Ibelin, of the Citadelian knights, negotiate an alliance with the Golden Concord on behalf of the Grand Dominion?”
- Three translation options: term logic, propositional logic, propositional logic where the atoms are categorical propositions.

Which language is “right”?

Whichever one allows you to represent the data that you have decided is important in a way that allows you derive a series of conclusions that answer the Q.

Coordinating language and the world

- Not just a modern concern
- Central to the Confucian idea of “rectification of names”:
 - ▶ rectification of names as a prerequisite for rectifying the world.
 - ▶ “the need to establish a universal correspondence between names and the realities they designate” (Indraccolo, “The ‘white horse is not horse’ debate,” 2017, p. 7)

Rectification of names

Kongfuzi (c.551–c.479 BCE, Confucius)

If language be not in accordance with the truth of things, affairs cannot be carried on to success. When affairs cannot be carried on to success, proprieties and music do not flourish. When proprieties and music do not flourish, punishments will not be properly awarded. When punishments are not properly awarded, the people do not know how to move hand or foot. Therefore a superior man considers it necessary that the names he uses may be spoken appropriately, and also that what he speaks may be carried out appropriately. What the superior man requires is just that in his words there may be nothing incorrect. (Analects, Book XIII, ch. 3, verses 4–7.)

Conclusion

The choices we make when doing logic are not neutral. They have an impact on the way we navigate the world and social order. We need to be prepared to justify and explain our choices!