Logic and the World: Some Reflections on the Relationship Between the Two

Sara L. Uckelman
Durham University
s.l.uckelman@durham.ac.uk
@SaraLUckelman

24 July 2026

What I say I want to teach my students

- What is logic?
- How to be a logician?

What I say I want to teach my students

- What is logic?
  - The study of good arguments
- How to be a logician?

What I say I want to teach my students

- What is logic?
  - The study of good arguments
  - Many different ways to define "good": Persuasive, deductively valid, epistemically sufficient, not fallacious, etc.
- How to be a logician?

What I say I want to teach my students

- What is logic?
  - The study of good arguments
  - Many different ways to define "good": Persuasive, deductively valid, epistemically sufficient, not fallacious, etc.
- How to be a logician?
  - What logical practice/logical research looks like.

What I *actually* want my students to learn (1)

- How to follow rules
- How to reason from a definition

What I *actually* want my students to learn (2)

Why?

Because the precision and clarity forced upon you when doing this will serve you well for the rest of your life.

Some examples (1)

Definition

Two categorical propositions are *contradictory* if the truth of one implies the falsity of the other, and vice versa.

Corollary

For any basic terms X and Y, $X\,a\,Y$ and $X\,o\,Y$ are contradictories, and $X\,e\,Y$ and $X\,i\,Y$ are contradictories.

Proof.

We prove only the first of these, leaving the other as an exercise for the reader.

We need to show that if $X\,a\,Y$ is true, then $X\,o\,Y$ must be false, and vice versa. Let I be an interpretation that makes $X\,a\,Y$ true, that is, $I \vDash X\,a\,Y$. It follows from the definition of truth that $I(Y) \subseteq I(X)$. Since $I(Y) \subseteq I(X)$, it is not true that $I(Y) \not\subseteq I(X)$. So it is not true that $I \vDash X\,o\,Y$. So $I \nvDash X\,o\,Y$, that is $X\,o\,Y$ is false on this interpretation.

Now, conversely, suppose that I is an interpretation that makes $X\,o\,Y$ true, that is $I \vDash X\,o\,Y$. It follows from the definition of truth that $I(Y) \not\subseteq I(X)$. Since $I(Y) \not\subseteq I(X)$, it is not true that $I(Y) \subseteq I(X)$. So it is not true that $I \vDash X\,a\,Y$. So $I \nvDash X\,a\,Y$, that is $X\,a\,Y$ is false on this interpretation, which is what we needed to show. □

Some examples (2)

Lemma

The natural deduction rule $\wedge I$ is sound with respect to classical propositional semantics.

Proof.

Let n be a line of a proof annotated with $\wedge\mathrm{I}, i, j$ for $i < n$ and $j < n$ and $i \neq j$. That is, the formula at line n is of the form $\phi \wedge \psi$, where ϕ occurs on line i and ψ occurs on line j. Assume that the formulas on lines i and j are true; that is, ϕ is true and ψ is true. It follows from the truth-table for $\wedge$ that $\phi \wedge \psi$ is also true. But that is the formula on line n. So the formula on line n is true. $\square$

Why are these so *hard*?

- It's just copying down the relevant clauses of the definitions
- It's *too simple*. Can it really be so simple? Yes!

Lesson # 1

Logic is easy!

Once you've got the definitions, proofs fall out of them for free!

. . . *once you've got the definitions* . . .

Lesson # 2

Logic is hard!

You've got to get the definitions right.

An object lesson in definitions

1. Are there statements which are neither true nor false?
2. Are there statements which are both true and false?
3. Is there a difference between truth and necessary truth?
4. Is there a difference between falsity and necessary falsity?
5. Can we argue about or with statements whose truth value we don't know?
6. Is there one correct mode of reasoning, or does correct reasoning vary depending on context?

What does it mean to get the definitions “right”? (1)

- right definitions get you easy proofs
- right definitions allow you to prove true things

What is needed is **appropriate coordination between the definitions and the world**

What does it mean to get the definitions "right"? (2)

Getting definitions "right" requires making decisions about what is important:

- What parts of the world will you highlight?
- What parts of the world will you sideline?

Is this argument valid? (1)

All mammals are animals.
All cats are mammals.
Therefore all cats are animals.

Fix a definition of validity: "An argument is valid if every model of the premises is also a model of the conclusion."

Is this argument valid? (2)

All mammals are animals.
All cats are mammals.
Therefore all cats are animals.

$A\ a\ M$	p	$\forall x(Mx \rightarrow Ax)$
$M\ a\ C$	q	$\forall x(Cx \rightarrow Mx)$
$A\ a\ C$	r	$\forall x(Cx \rightarrow Ax)$
Y	N	Y

What about this argument?

All mammals are animals.
All cats are mammals.
Therefore some cat is an animal.

$A\ a\ M$	p	$\forall x(Mx \rightarrow Ax)$
$M\ a\ C$	q	$\forall x(Cx \rightarrow Mx)$
$A\ i\ C$	r	$\exists x(Cx \wedge Ax)$
Y	N	N

Validity is not constant

How we define our language, how we define our models determines which translations are valid and which are not.

What does this tell us about our original argument?

Why definitions matter

- Getting your language right determines what is valid, what you can express, what you can prove.
- Decisions make evident what you think is relevant/important/valuable/worth tracking.

These decisions are not neutral!

An object lessons for my students

- Take-home mid-term exam
- ˜ 1.5 pages of prose and one question to answer: “Should Captain Aenne Qadu d’Ibelin, of the Citadelian knights, negotiate an alliance with the Golden Concord on behalf of the Grand Dominion?”
- Three translation options: term logic, propositional logic, propositional logic where the atoms are categorical propositions.

Which language is “right”?

Whichever one allows you to represent the data that you have decided is important in a way that allows you derive a series of conclusions that answer the Q.

Coordinating language and the world

- Not just a modern concern
- Central to the Confucian idea of "rectification of names":
  - rectification of names as a prerequisite for rectifying the world.
  - "the need to establish a universal correspondence between names and the realities they designate" (Indraccolo, 2017, p. 7)

Rectification of names

Kongfuzi (c.551–c.479 BCE, Confucius)

> *If language be not in accordance with the truth of things, affairs cannot be carried on to success. When affairs cannot be carried on to success, proprieties and music do not flourish. When proprieties and music do not flourish, punishments will not be properly awarded. When punishments are not properly awarded, the people do not know how to move hand or foot. Therefore a superior man considers it necessary that the names he uses may be spoken appropriately, and also that what he speaks may be carried out appropriately. What the superior man requires is just that in his words there may be nothing incorrect. (Analects, Book XIII, ch. 3, verses 4–7.)*

Xunzi (c.310–c. after 238 BCE): synthesiser of Confucianism with Daosim and Mohist thought.

Lesson #3

The choices we make when doing logic are not neutral. They have an impact on the way we navigate the world and social order. We need to be prepared to justify and explain our choices!