

The Quest for Logical Consequence in the Middle Ages

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Introduction

- Where this talk comes from
- Logical consequence for Tarski
- Logical consequence for Aristotle
- Logical consequence in the Middle Ages
- *Obligationes*
- Conclusion

Truth tables (1)

φ	$\neg\varphi$		φ	ψ	$\varphi \wedge \psi$
T	F		T	T	T
F	T		T	F	F
			F	T	F
			F	F	F

φ	ψ	$\varphi \vee \psi$		φ	ψ	$\varphi \rightarrow \psi$
T	T	T		T	T	T
T	F	T		T	F	F
F	T	T		F	T	T
F	F	F		F	F	T

Truth tables (2)

- Fundamental starting point of every intro logic class.
- Shortened truth-table method for testing for tautologies:
- Every argument is the conjunction of its premises implying its conclusion.
- Correspondence between tautologies and valid arguments.

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It's all so obvious!

Logical consequence for Tarski (1)

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[logical consequence after Tarski is] no longer seen as the result of conceptual analysis—when the need for analysis is forgotten, and the definition is treated as common knowledge. . . [5, p. 1].

Logical consequence for Tarski (2)

Tarski's work is often treated as the first real solution to the problem, a problem that "confronted early, formal logicians" whose work was "driven by an interest in the intuitive notions of logical truth and logical consequence, but the only precise access to these notions was through specific, proof-theoretic characterizations" [5, p. 5].

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All M are P	No M is P	All M are P	No M is P
All S are M	All S are M	Some S is M	Some S is M
All S are P	No S is P	Some S is P	Some S is not P

Barbara

Celarent

Darii

Ferio

Logical consequence for Aristotle (2)

- Identify good rules of transformation: simple conversion, accidental conversion, transposition, reductio:

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Swap the subject
and predicate
in *i* and *e* claims

Simple

Swap subject
and predicate
and change from *a* to *i*
and *e* to *o*

Accidental

Take the contradictory
of the conclusion
and derive a contradiction
with the premises

RAA

Logical consequence for Aristotle (3)

- Demonstrate that some syllogisms can be reduced to the obviously good ones via these rules, and give counter examples of all ones that can't be:

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- Demonstrate that some syllogisms can be reduced to the obviously good ones via these rules, and give counter examples of all ones that can't be:

e.g., Fesapo (“No P are M , all M are S , therefore some S is not P ”):

- Simply convert first premise to “No M are P ”.
- Accidentally convert second premise to “Some S is M ”.
- Use Ferio to derive the conclusion “Some S is not P ”.

Logical consequence for Aristotle (4)

- None of this has anything to do with validity (in the modern sense)!
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- It's also quite narrow/constricted. No *general* notion of “consequence”.
- When does “*valet*” first turn up?
- In the Middle Ages. . . and not obviously about *validity* rather than *soundness*.

Logical consequence in the Middle Ages (1)

Our motivating question: So what does logical consequence look like *after* we go beyond Aristotle but *before* we absorb Tarski?

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How do we figure out what logical consequence *is* before it is obvious what it is?

Logical consequence in the Middle Ages (2)

Medieval treatises on consequences (late 13th/early 14th C):

- “*consequentiae*” covers variously: entailment, inference, argument, syllogism, and (true) conditional.
- “*antecedens*” and “*consequens*”: What comes before and what follows after.
- *consequens/consequentia/consequenter*: Fundamentally a *semantic* notion.
- Investigations into the notion of “what follows from what”.
- How do we know when one proposition “follows from” another?

The key driver in the development of a concept of logical consequence in the Middle Ages was the theory of *obligationes*.

Obligationes (1)

Definition

An *obligatio* is a turn-based disputation between two agents, the Opponent and the Respondent, where the Opponent puts forward a sequence of propositions, and the Respondent is obligated to follow certain rules in his responses (accept/concede, deny/reject, doubt) to the Opponent's propositions.

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The *obligatio* continues until the Opponent calls "*Cedat tempus*" ("Time's up"), whereupon the responses of the Respondent are analysed with respect to the rules the Respondent was supposed to follow, to determine whether the Respondent has responded well or badly.

Obligationes (2)

- Many different variants of this type of disputation exist (*positio*, *depositio*, *dubitatio*, *petitio*, *impositio* or *institutio*, *rei veritatis* or *sit verum*).
- Examples of obligational-style reasoning are often found in treatises on *sophismata* and *consequentiae*.
- Many modern commentators: Role and purpose is often unclear. (I dispute this).

Some authors who wrote on *obligationes*.

- Early anonymous treatises dating from the late 12th/early 13th century.
- William of Sherwood (1190–1249).
- Nicholas of Paris (fl. 1250).
- Walter Burley (c. 1275–1344).
- Roger Swyneshed (d. 1365).
- Richard Kilvington (d. 1361).
- William Ockham (c. 1285–1347).
- Robert Fland (c. 1350).
- John of Holland (1360s).
- Richard Brinkley (temp. 1365–1370).
- Richard Lavenham (d. 1399).
- Ralph Strode (d. 1387).
- Peter of Candia (late 14th C).
- Peter of Mantua (d. 1399).
- Paul of Venice (c. 1369–1429).

Obligationes: The basics (1)

Standard treatment of *positio* from Walter Burley's *De obligationibus*, c1302:

The opponent's job is to use language in a way that makes the respondent grant impossible things that he need not grant because of the positum. The respondent's job, on the other hand, is to maintain the positum in such a way that any impossibility seems to follow not because of him but rather because of the positum [3, 4].

⇒ the goal is consistency, not logical truth.

Obligationes: The basics (2)

- General Rule 1** Everything following from an *obligatum* must be granted (where '*obligatum*' is interpreted as what has been granted or what must necessarily be granted).
- General Rule 2** Everything incompatible with the *obligatum* must be denied.
- General Rule 3** One must reply to what is irrelevant in accordance with its own quality.

Definition

A proposition is *irrelevant* or *impertinent* if neither it nor its negation follows from the set of propositions which have already been conceded (which includes the negations of propositions which have been denied).

Positio according to Burley

Rule

Everything that is posited and put forward in the form of the positum during the time of the *positio* must be granted.

Rule

Everything that follows from the positum must be granted. Everything that follows from the positum either together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be granted.

Rule

Everything incompatible with the positum must be denied. Likewise, everything incompatible with the positum together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be denied.

An example *positio*

Suppose φ does not imply $\neg\psi$ and φ is known to be false.

	Opponent	Respondent
1	φ .	I admit it.
2	$\neg\varphi \vee \psi$.	I grant it.
3	ψ	I grant it.

Note

This example shows how, given a *positum* which is false, but not inconsistent, the Opponent can force the Respondent to concede any other consistent proposition.

A more interesting example

Opponent
1 φ or ψ must be granted.

Respondent
I admit it.

A more interesting example

Opponent

- 1 φ or ψ must be granted.
- 2 φ must be granted.

Respondent

I admit it.
I deny it.

A more interesting example

	Opponent	Respondent
1	φ or ψ must be granted.	I admit it.
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A more interesting example

	Opponent	Respondent
1	φ or ψ must be granted.	I admit it.
2	φ must be granted.	I deny it.
3	φ follows from the positum and the opposite of something correctly denied	I grant it.
4	φ must be granted.	??

Obligationes: Aristotelian roots (1)

Burley's definition of the goal of *obligationes* is a nearly exact quotation from Book VIII, chapter 4 of the *Topics*, where Aristotle distinguishes three types of disputations:

- 1 disputations for teaching and learning (*didactic*),
- 2 disputations for competitive purposes (*eristic* or *contentious*),
- 3 disputations for the sake of practice and experiment (*dialectic*).
- 4 (In *Sophistical Refutations* ch. II, Aristotle adds "examinational" disputations).

Obligations: Aristotelian roots (2)

Aristotle on question-and-answer games:

With regard to the giving of answers, we must first define what is the business of a good answerer, as of a good questioner. The business of the questioner is so to develop the argument as to make the answerer utter the most implausible of the necessary consequences of his thesis; while that of the answerer is to make it appear that it is not he who is responsible for the impossibility or paradox, but only his thesis (Topics) [2, p. 268].

Obligations: Aristotelian roots (3)

Two common motivating principles:

- 1 “from the possible nothing impossible follows” (necessary but not sufficient for defining logical consequence)
- 2 the possible is “that which is not necessary but, being assumed, results in nothing impossible” (defining a term in the previous)

Just as we say that something possible must be conceded in order to see what follows from it, similarly we have it from Aristotle that something impossible must be conceded in order to see what happens then” [1, p. 217].

Aristotelian not Aristotle. The most similar statement is found in Boethius's *De Hypotheticis Syllogismis*, attributed to Eudemus.

Obligations: Aristotelian roots (4)

Combine these two principles with the instruction that “something possible must be conceded in order to see what follows from it”, we have a mechanism to determine what follows from what.

Arguing for my thesis

Ways things can go wrong:

- You can make a (logical) mistake
- There's a hidden contradiction

Unsurprising, then, that *obligationes* treatises gave way to two new genres of treatise, on *consequentiae* and *sophismata*!

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Determining which has happened is a way of zeroing in on *what follows from what* \rightsquigarrow an **empirical** investigation of consequence by trial and error.

Conclusions

- When you already know what logical consequence is, it's obvious.
- Before you know what logical consequence is, it's baffling.
- Logical consequence or “what follows from what” is an *empirical* matter, which can be investigated *empirically*.
- The medieval method of doing this is *obligationes*.

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