

About the Future / About the Past: Diodorus's Master Argument

Sara L. Uckelman
Durham University
s.l.uckelman@durham.ac.uk
@SaraLUckelman [twitter/bsky]

Is, Ought, and Other Barriers to Entailment
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What is an entailment barrier?

Gillian Russell, *Barriers to Entailment: Hume's Law and Other Limits on Logical Consequence*:

A barrier to entailment is something that gets in the way of there being valid arguments from premises of one kind to conclusions of another kind [p. 1].

Examples

- The particular/universal barrier: No universal claims from particular ones (Aristotle, B. Russell)
- The past/future barrier: No claims about the future from claims about the past
- The is/ought barrier: No claims about how things ought to be from claims about how things are. (Anscombe, Flew, Hare, Hume, Jaggar, Karmo, von Kutschera, Nowell-Smith, Pigden, Popper, Prior, Schurz, Shorter, Thomason & Thomason)
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Russell's thesis: Theorems asserting the existence of barriers like these can all be generated/proven in a uniform way.

Characterizing classes of sentences

In each case we started with a logic—in the sense of a language with a set of models used to define an entailment relation on that language—and identified a binary relation (such as model extension, or future-switching) on those models. This relation was used to define sentences, and sets of sentences, which were fragile or anti-fragile with respect to it [p. 113].

About “aboutness”

- Russell uses the notions of fragility and breakability to characterize when a sentence is *genuinely* “about” something, e.g., “about the past,” “about universality,” “about normativity.”
- This is a *semantic* notion, not a syntactic one: a sentence which involves no universal quantifier may yet nevertheless be *about* universality, e.g., $\neg\exists xFx$ turns out to be a universal sentence even if there is no universal quantifier [p. 70].

Can we use this notion of “aboutness” to make sense of the Master Argument of Diodorus Cronus?”

What is the Master Argument?

- Found in Epictetus's (c50–c135CE) *Discourses* Book II, Chapter 19.
- Calling the κυριεύων λογος, translated the “Master Argument”; could be called the “Ruling Argument”.
- The joint inconsistency of three statements.
- Diodorus Cronus (c340–c284 BCE) argued from the truth of the first two to the falsity of the third.

The first statement

Modern discussions of the argument turn on the difficulty of translating the three statements, particularly the first.

Statement 1

- Everything true as event in the past is necessary. [Oldfather]
- Every proposition true about the past is necessary. [Mates]
- Everything past is necessarily true. [Carter]
- Everything that is past and true is (now) necessary. [Rescher]
- Every true proposition about the past is necessary. [Prior]

The second and third statements

Statement 2

- An impossible does not follow a possible. [Oldfather]
- An impossibility is not the consequence of a possibility. [Carter]
- An impossible proposition cannot follow from (or after) a possible one. [Mates]
- An impossible proposition never follows from a possible one. [Prior]

Statement 3

- What is not true now and never will be, is nevertheless possible. [Oldfather]
- What neither is nor will be true, is nevertheless possible. [Prior]
- There is a proposition which is possible, but which neither is nor will be true. [Mates]

Prior's reconstruction (1955)

How does one get from Statements 1 and 2 to the negation of Statement 3?

(Translating from Polish notation to infix notation, and taking $\neg\Diamond\neg$ to be \Box)

- $Pp \rightarrow \Box Pp$ (Statement 1)
- $\neg\Diamond q \rightarrow (\Box(p \rightarrow q) \rightarrow \neg\Diamond p)$ (Statement 2)
- $p \rightarrow HFP$ (“When anything is the case, it has always been the case that it will be the case”).
- $(\neg p \wedge \neg Fp) \rightarrow P\neg Fp$ (“When anything neither is nor will be the case, it has been the case that it will not be the case.”)

(Prior adds (3) and (4) arguing that they are “likely to have been taken for granted both by Diodorus and by his main opponents.”)

The Master Argument as an argument for determinism (1)

The denial of [Statement 3] is equivalent to the view that if a proposition is possible, then either it is true now or it will be true at some future time. So in a nutshell the argument is that an event which never will happen and is not happening now cannot be possible, and hence everything happening now or in the future is necessary [Øhrstrøm and Hasle, p. 28]

The Master Argument as an argument for determinism (2)

It's even worse if we take our atomic propositions to be *temporally-definite*, i.e., containing an explicit time reference.

If p is temporally definite, then we have not only this simplified version of Statement 1:

$$Pp \rightarrow \Box p$$

we also have the “unpalatably deterministic thesis” [Rescher & Urquhart, p. 195]:

$$p \rightarrow \Box p$$

that is, the Leibnizian principle “*unumquodque, quando est, oportet esse*”.

Øhrstrøm and Hasle's diagnosis

Recall Prior's formalization of Statement 1 "Every true proposition about the past is necessary":

$$Pp \rightarrow \Box Pp$$

Øhrstrøm and Hasle's diagnosis

Recall Prior's formalization of Statement 1 “Every true proposition about the past is necessary”:

$$Pp \rightarrow \Box Pp$$

Just because a sentence has a P operator in it, doesn't mean it's *actually about the past*. Hence, we should restrict our attention only to propositions which are genuinely *about the past*:

In our reconstruction, we do not have to assume any more than the necessity of propositions which are genuinely about the past [Øhrstrøm and Hasle, p. 29].

Can Russell's notion of "aboutness" help us demarcate which propositions are genuinely about the past?

When is a sentence genuinely about the past?

We will say that Past sentences are future-switch anti-fragile which means that whenever they are true in a model, they are true in all future-switches of that model. . . An example is Pp . If it is true in a model, there is some $t < n$ where p is true. Since that feature will be preserved through future-switches, Pp will be true in any future-switch of the model. So the sentence is classified as Past, as one would expect [p. 91].

Some conceptual background

- Basic tense logic with forward and backward modalities,
- interpreted over models with a designated point n (“now”), where the ordering relation is
 - ▶ transitive,
 - ▶ anti-symmetric
 - ▶ irreflexive
 - ▶ (dense, R-total and L-total, R-extendible and L-extendible. . .)
- and truth-in-a-model is defined as truth at the designated point.
- future-switching: “one model is a future-switch of another if you can get from the first to the second by *changing what happens in the future*. The underlying idea is that future-sentences can be made false by changing what happens in the future, whereas (present and) past sentences cannot” [p. 87].

Future-switching, precisely

Definition (Basic Future-Switching (Υ))

Let $\mathfrak{M}, \mathfrak{N}$ be temporal models. \mathfrak{N} is a *basic future-switch* of \mathfrak{M} ($\mathfrak{M} \Upsilon \mathfrak{N}$) if:

- $T_{\mathfrak{N}} = T_{\mathfrak{M}}$
- $<_{\mathfrak{N}} = <_{\mathfrak{M}}$
- $n_{\mathfrak{N}} = n_{\mathfrak{M}}$
- and for all atoms p and $t \in T_{\mathfrak{N}}$ such that $t \leq n$,

$$I_{\mathfrak{N}}(p, t) = I_{\mathfrak{M}}(p, t)$$

That is, the two models “differ only in the values they assign to sentence letters relative to future times” [p. 88].

Back to Statement 1 and determinism

The statement 'It has been that Dion never will be here,' (in symbols: $P\neg Fq$) should not be counted as necessary even if it is true. . . there is no a priori reason to exclude the conceptual possibility of Dion's being here at some future time, or his 'having always been going to be here'. [Øhrstrøm and Hasle, p. 29].

That is, even though $P\neg Fq \rightarrow \Box P\neg Fq$ is a substitution instance of $Pp \rightarrow \Box Pp$, we want to accept the latter without accepting the former.

Is $P \rightarrow Fq$ about the past?

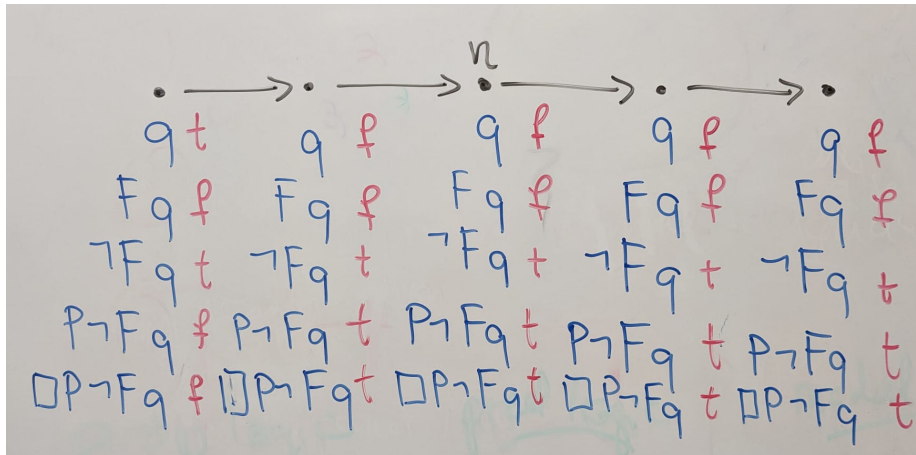
- Rephrase according to Russell: Is $P \rightarrow Fq$ future-switch anti-fragile?
- That is, is the case that whenever $P \rightarrow Fq$ is true in a model, it is true in all future-switches of that model?

What would this give us?

Not so much a barrier theorem in the sense of Russell, but a limitation: We can avoid the unpalatable deterministic consequences of the Master Argument by restricting uniform substitution on Statement 1: $Pp \rightarrow \Box Pp$. This is not to be taken as an axiom, but rather a schema where every substitution instance $P\phi \rightarrow \Box P\phi$ is admitted *provided that ϕ is genuinely about the past, that is, ϕ is future-switch anti-fragile.*

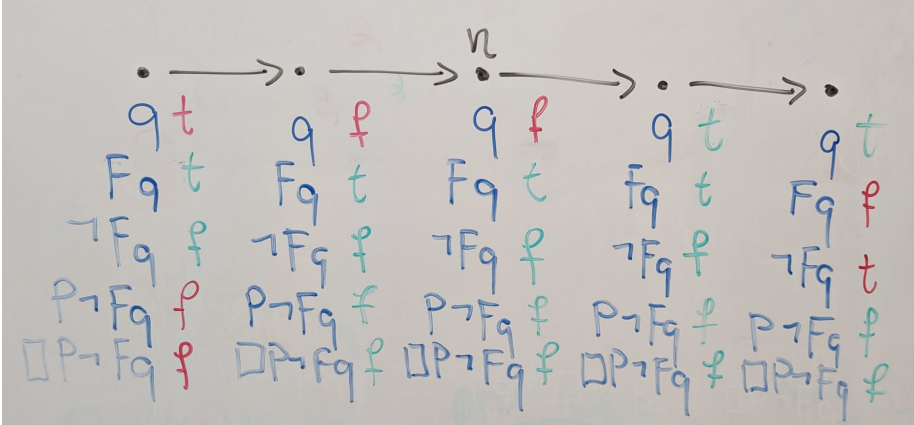
Let's find out! (1)

$P \rightarrow Fq$ is true on this model (at n , transitive arrows not drawn, assuming necessity is, in addition, reflexive):



Let's find out! (2)

If $P\neg Fq$ is future-switch anti-fragile, then it should be impossible to create a future-switch of this model where $P\neg Fq$ is false (at n). But we can!



Let's find out! (3)

So, $P\neg Fq$ is not future-switch anti-fragile, so it is not about the past, despite having a P operator, so we shouldn't allow it as a substitution instance.

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So, $P\neg Fq$ is not future-switch anti-fragile, so it is not about the past, despite having a P operator, so we shouldn't allow it as a substitution instance.

Yay! We've solved it! ...or have we?

Some initial conclusions/confusions

- 1 We can create such models, but on these models, $\Box P \rightarrow Fq$ is also false, which makes Statement 1 still turn out to be true, albeit vacuously. What about our intuition that the problem with things like 'It has been that Dion never will be here' is that the antecedent can be *true* without the consequent?

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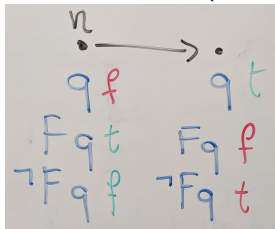
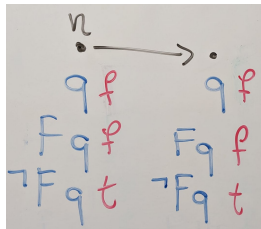
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- 2 What is being substituted? If we take $Pp \rightarrow \Box Pp$ as being either a *schema*, or an axiom, then it is p that we substitute into, not Pp . So shouldn't we be considering the future-switch anti-fragility of $\neg Fq$ instead of $P \neg Fq$?

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- 3 $\neg Fq$ is also not future-switch anti-fragile! (As we'd expect.)
- 4 But (going back to (2)) p itself doesn't have to be about the past—it becomes relevant to Statement 1 **just as soon as we prepend P to it**. So surely it's $P\phi$ that we need to evaluate whether it is genuinely about the past or not.

Work in progress. . .

It still seems to me that there should be something that can be said here, but I'm not sure what, and I would appreciate very much your. . .

... Questions!
(and... Thank you!)