

# *Obligations: A Method for Analyzing Paradoxes*

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# What is an *obligatio*?

## Definition

An *obligatio* is a turn-based disputation between two agents, the Opponent and the Respondent, where the Opponent puts forward a sequence of propositions, and the Respondent is obligated to follow certain rules in his responses (accept/concede, deny/reject, doubt) to the Opponent's propositions.

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The *obligatio* continues until the Opponent calls "*Cedat tempus*" ("Time's up"), whereupon the responses of the Respondent are analysed with respect to the rules the Respondent was supposed to follow, to determine whether the Respondent has responded well or badly.

# Obligationes

- Many different variants of this type of disputation exist (*positio*, *depositio*, *dubtatio*, *petitio*, *impositio*, *rei veritatis*).
- Role and purpose is often unclear.
- Examples of obligational-style reasoning are often found in *sophismata* and *insolubilia* texts.
- The disputation rules are used as a meta-logic for reasoning about the *sophismata* and *insolubilia* sentences.

## Some authors who wrote on *obligationes*.

- William of Sherwood (1190–1249).
- Nicholas of Paris (fl. 1250).
- Walter Burley (*or* Burleigh) (c. 1275–1344).
- Roger Swyneshed (d. 1365).
- Richard Kilvington (d. 1361).
- William Ockham (c. 1285–1347).
- Robert Fland (c. 1350).
- John of Holland (1360s).
- Richard Brinkley (temp. 1365–1370).
- Richard Lavenham (d. 1399).
- Ralph Strode (d. 1387).
- Peter of Candia (late 14th C).
- Peter of Mantua (d. 1399).
- Paul of Venice (c. 1369–1429).

## *Obligationes* according to Burley (1).

Walter Burley, *De obligationibus*, c.1302 [3, 2]. Burley defines the general goal of an *obligatio* as follows:

*The opponent's job is to use language in a way that makes the respondent grant impossible things that he need not grant because of the positum. The respondent's job, on the other hand, is to maintain the positum in such a way that any impossibility seems to follow not because of him but rather because of the positum.*

⇒ the goal is consistency, not logical truth or validity.

## *Obligationes* according to Burley (2).

- General Rule 1** Everything following from an *obligatum* must be granted (where '*obligatum*' is interpreted as what has been granted or what must necessarily be granted).
- General Rule 2** Everything incompatible with the *obligatum* must be denied.
- General Rule 3** One must reply to what is irrelevant in accordance with its own quality.

### Definition

A proposition is *irrelevant* or *impertinent* if neither it nor its negation follows from the set of propositions which have already been conceded (which includes the negations of propositions which have been denied).

## *Positio* according to Burley.

### Rule

Everything that is posited and put forward in the form of the positum during the time of the *positio* must be granted.

### Rule

Everything that follows from the positum must be granted. Everything that follows from the positum either together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be granted.

### Rule

Everything incompatible with the positum must be denied. Likewise, everything incompatible with the positum together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be denied.



## Possibility, paradox, and *obligationes*

Medieval authors often cited two Aristotelian principles:

- “from the possible nothing impossible follows”
- the possible is “that which is not necessary but, being assumed, results in nothing impossible”

E.g., the anonymous *Tractatus Emmeranus*:

*Just as we say that something possible must be conceded in order to see what follows from it, similarly we have it from Aristotle that something impossible must be conceded in order to see what happens then [1, p. 217].*

## An example *positio*.

Suppose  $\varphi$  does not imply  $\neg\psi$  and  $\varphi$  is known to be false.

	<b>Opponent</b>	<b>Respondent</b>
1	$\varphi$ .	I admit it.
2	$\neg\varphi \vee \psi$ .	I grant it.
3	$\psi$	I grant it.

### Note

This example shows how, given a *positum* which is false, but not inconsistent, the Opponent can force the Respondent to concede any other consistent proposition.

## A more interesting example

**Opponent**  
1  $\varphi$  or  $\varphi$  must be granted.

**Respondent**  
I admit it.

## A more interesting example

### Opponent

- 1  $\varphi$  or  $\psi$  must be granted.
- 2  $\varphi$  must be granted.

### Respondent

I admit it.  
I deny it.

## A more interesting example

	<b>Opponent</b>	<b>Respondent</b>
1	$\varphi$ or $\psi$ must be granted.	I admit it.
2	$\varphi$ must be granted.	I deny it.
3	$\varphi$ follows from the positum and the opposite of something correctly denied	I grant it.

## A more interesting example

	<b>Opponent</b>	<b>Respondent</b>
1	$\varphi$ or $\psi$ must be granted.	I admit it.
2	$\varphi$ must be granted.	I deny it.
3	$\varphi$ follows from the positum and the opposite of something correctly denied	I grant it.
4	$\varphi$ must be granted.	??

## *Positio* according to Paul of Venice

*De obligationibus*, Part II, Tract. 8 of the *Logica Magna* [6]:

### Definition

An *obligatio* is a relation limiting one to uphold some statement, or its equiform, in some way . . . It is based on the obligator, by virtue of the *positio* or the *depositio*; and on the obligated, by reason of his *admissio*. [6, pp. 7, 11].

The *positum* should be admitted if there is no impediment arising in doing so [6, pp. 48–49]; any proposition [6] once conceded must be conceded if it is ever put forward again [6, pp. 34–35]; relevant propositions must be conceded if they follow and denied if they do not [6, pp. 54–63]; irrelevant propositions must be conceded if they are known to be true, denied if known to be false, and doubted if neither [6, pp. 64–65].

### Definition (Relevance)

A proposition  $\varphi$  is relevant to a proposition  $\psi$  if  $\varphi$  follows from it or is inconsistent with  $\psi$ ; it is irrelevant otherwise [6, pp. 24–25].

More specifically,  $\varphi$  could be relevant:

- to the positing of the *positum*
- to the *positum* itself [Swyneshed]
- to both together, or
- to the *positum* taken together with correctly granted propositions and [the opposite of] correctly denied propositions [6, pp. 24–29]. [Burley]



## A paradox (?)

Can something known to you be uncertain to you or not known by you?

Yes!

Let's set the stage:

*(1) I assume (a) that you know that A is one of the two propositions 'God exists' and 'A human being is a donkey', (b) that one A is every A, and (c) that it is hidden from you which of the propositions is A, but (d) you know perfectly well that the proposition 'God exists' is necessary and the other, 'A human being is a donkey', impossible [5, p. 3].*

*(2) Every proposition you consider which you do not know to be true and do not know to be false is uncertain to you [5, p. 5].*

# Using obligational reasoning to reason about this paradox (1)

“You could work out from this how to reply in the case under discussion.

For example:

- |    |                             |                 |
|----|-----------------------------|-----------------|
| 1  | $A$ is true.                | I am uncertain. |
| 2  | $A$ is false.               | I am uncertain. |
| 3  | $A$ is contingent.          | I deny it.      |
| 4  | $A$ is possible.            | I am uncertain. |
| 5  | $A$ is necessary.           | I am uncertain. |
| 6  | $A$ is impossible.          | I am uncertain. |
| 7  | You know that $A$ is true.  | I deny it.      |
| 8  | You know that $A$ is false. | I deny it.      |
| 9  | $A$ is uncertain to you.    | I deny it.      |
| 10 | $A$ is known by you.        | I am uncertain. |

## Using obligational reasoning to reason about this paradox (2)

- |    |   |                 |
|----|---|-----------------|
| 11 | You know that $A$ is known by you.          | I deny it.      |
| 12 | You are uncertain that $A$ is known by you. | I grant it.     |
| 13 | You know that $A$ is uncertain to you.      | I deny it.      |
| 14 | Of $A$ you know that it is known by you.    | I am uncertain. |
| 15 | You are uncertain that $A$ is true.         | I grant it.     |
| 16 | Of $A$ I am uncertain that it is true.      | I deny it.      |
| 17 | About $A$ I am uncertain.                   | I deny it.      |
| 18 | You are uncertain about $A$                 | I deny it.      |

Note re: 16 and 17: It is unclear to me whether this is a genuine mistake, and this should read “Of  $A$  you are uncertain that it is true”, i.e.,  $O$  is saying this sentence to  $R$  and is speaking about  $R$ ’s uncertainty, or if Paul genuinely intended “I”, in which case this would have to be read as  $O$  putting forward a proposition about *his own* uncertainty. In what follows, I have adopted the latter reading.

## Using obligational reasoning to reason about this paradox

(3)

- |    |   |                |
|----|---|----------------|
| 19 | Of something true you know that it is <i>A</i> .                            | I deny it.     |
| 20 | Of <i>A</i> you know that it is something.                                  | I grant it.    |
| 21 | You know that <i>A</i> is <i>A</i> .  | I grant it.    |
| 22 | Of <i>A</i> you know that it is <i>A</i> .                                  | I deny it.     |
| 23 | Of <i>A</i> you know that it is <i>A</i> or something other than <i>A</i> . | I grant it.    |
| 24 | Of <i>A</i> you know that it is something other than <i>A</i> .             | I deny it.     |
| 25 | Of <i>A</i> you know that it is true or false.                              | I grant it.    |
| 26 | You know that <i>A</i> is necessary or impossible.                          | I grant it.    |
| 27 | Of <i>A</i> you know that it is possible or contingent.                     | I am uncertain |
| 28 | Of <i>A</i> you know that it is impossible or contingent.                   | I am uncertain |
| 29 | You know that <i>A</i> is possible or contingent.                           | I deny it.     |
| 30 | You know that <i>A</i> is contingent or impossible.                         | I deny it.     |

And so on for innumerable other propositions, some of which should be granted, some of which should be denied, and about some of which one should say that one is uncertain, if one wants to examine the matter carefully [5, pp. 19, 21].

## Using obligational reasoning to reason about this paradox

While he does not say explicitly that this is an obligational disputation, nor make explicit the rules involved, it is clear from the presentation—of propositions which the reader is responding to either by conceding, denying, or doubting—and from the initial assumptions that this is a type of *dubitatio*, namely about the nature of  $\hat{a}$ . Thus, starting from an initial model where R is uncertain about whether the sentence is “God exists” or “A human being is a donkey,” if he correctly follows standard rules for *dubitatio*<sup>1</sup>, R should never be forced into admitting that he knows the sentence (cf. [7, Theorem 24]).

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<sup>1</sup>Such as those given by Nicholas of Paris, cf. [8, §4.3].

## *Dubitatio.*

In *dubitatio*, the **respondent** must **doubt** the statement that the **opponent** puts forward (called the *dubitatum*) [9, 7, 4].

- Some authors (later medieval and modern) call *dubitatio* a trivial variant of *positio*, like *depositio*. But it isn't:
  - ▶ In *positio*, doubt can always be ungraded to concession or denial.
  - ▶ The primary obligation is to doubt a statement which is known to be true (or false).
  - ▶ Many different ways to create doubt from knowledge.
  - ▶ Ordinary knowledge in epistemic logic is irrevocable.

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