Christine Ladd-Franklin and the Syllogistic Problem She Solved

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The contribution of Christine Ladd-Franklin (Russinoff)

In 1883, while a student of C.S. Peirce at Johns Hopkins University, Christine Ladd-Franklin published a paper titled On the Algebra of Logic, in which she develops an elegant and powerful test for the validity of syllogisms that constitutes the most significant advance in syllogistic logic in two thousand years... In this paper, I bring to light the important work of Ladd-Franklin so that she is justly credited with having solved a problem over two millennia old [Russinoff, 1999, p. 451, emphasis added].

The contribution of Christine Ladd-Franklin (Pietarinen)

The result [of LF's thesis] was the ground-breaking discovery involving the reduction of Aristotelian syllogistics into a single formula [Pietarinen, 2013, p. 3, emphasis added] [preprint] / The result was the reduction of the Aristotelian syllogistics into a single formula" [Pietarinen, 2013, p. 142] [published].

See Russinoff (1999) on how, in her dissertation, Ladd-Franklin in fact managed to solve — or at least to see the solution to — the problem that was over two millennia old, though she did not give, nor could she have given the proof in such a rigorous form that is possible nowadays in the semantic terms of possible interpretations in varying domains" [Pietarinen, 2013, fn. 6].

What is the problem?

Everyone assumes we know what "the problem" is!

The Solution

Theorem

The argument of inconsistency,

$$(a \,\overline{\vee}\, b)(\bar{b} \,\overline{\vee}\, c)(c \vee a)\overline{\vee} \tag{II}$$

is the single form to which all the ninety-six valid syllogisms (both universal and particular) may be reduced [Ladd, 1883, p. 40].

Proof.

Any given syllogism is immediately reduced to this form by taking the contradictory of the conclusion, and by seeing that the universal propositions are expressed with a negative copula and particular propositions with an affirmative copula [Ladd, 1883, p. 40]. □

Reprise: What is the “form” of a syllogism?

- Is it “figure”?
- Is it “mood”?
- Is it something else?

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  Hardly likely.

What did Ladd think she was doing?

According to Ladd, her algebra, including the antilogism, "contains a solution of what Mr. Jevons calls the 'inverse logical problem'" [Ladd, 1883, p. 50]. The *Inverse Problem* is:

> *given certain combinations inconsistent with conditions to determine those conditions [Jevons, 1880, p. 252].*

The Inverse Logical Problem

A more precise characterisation of the *Inverse Logical Problem Involving Three Terms* is given in [Jevons, 1874, p. 157]:

> *Three terms and their negatives may be combined... in eight different combinations, and the effect of laws or logical conditions is to destroy any one or more of these combinations. Now we may make selections from eight things in* 2^8 *or 256 ways; so that we have no less than 256 different cases to treat, and the complete solution is at least fifty times as troublesome as with two terms... The test of inconsistency is that each of the letters* A, B, C, a, b, c *shall appear somewhere in the series of combinations; but I have not been able to discover any mode of calculating the number of cases in which inconsistency would happen... an exhaustive examination of the combinations in detail is the only method applicable [Jevons, 1874, pp. 157–158].*

Solving the Inverse Logical Problem (1)

The solution that Jevons provides "consists in inventing laws and trying whether their results agree with those before us" [Jevons, 1880, p. 252].

Both Schröder and Boole also tried to solve the problem:

> *The task which Boole accomplished was the complete solution of the problem:—given any number of statements, involving any number of terms mixed up indiscriminately in the subjects and the predicates, to eliminate certain of those terms, that is, to see exactly what the statements amount to irrespective of them, and then to manipulate the remaining statements so that they shall read as a description of a certain other chosen term (or terms) standing by itself in a subject or predicate [Ladd Franklin, 1889, p. 543].*

Solving the Inverse Logical Problem (2)

Though Boole's rule gets the conclusion right, Ladd is nevertheless critical of it. She quotes Venn, who said that Boole's method was "a terribly long process" [Ladd, 1883, p. 50], more theoretical than practical. Further than that, Boole's form of the conclusion would have to be altered to fit the notation of her algebra, and is also "not that which is most natural or most frequently useful" [Ladd, 1883, p. 50]. This is because it is "suited only to a logic of extension" and does not work well under an intensive interpretation [Ladd, 1883, p. 50].

The Five Algebras of Logic

Ladd identifies five algebras of logic, due to:

- Boole (in *Laws of Thought*)
- Jevons [Jevons, 1864]
- Schröder [Schröder, 1877]
- McColl [McColl, 1877]
- Peirce [Peirce, 1867]

Her aim was to introduce a sixth that addresses what she sees as are the drawbacks of the previous attempts.

Terms

The basic component of Ladd's algebras is subject and predicate terms. Atomic subject and predicate terms (hereafter simply called 'terms') are indicated by, e.g., a, b, c. Ladd follows Wundt and Peirce in using ∞ as a term to represent the domain of discourse [Ladd, 1883, p. 19].

Complex terms can be formed from atomic terms as follows:

- $\bar{a} =$ "what is not a".
- $a \times b =$ "what is both a and b".
- $a + b =$ "What is either a or b".

At times, $a \times b$ will be represented as ab. Infinite series of $\times$ or $+$, or combinations of the two, are allowed. $\overline{\infty}$ is given its own symbol: 0 [Ladd, 1883, p. 19].

What can be done with terms

- The identity of the subject and predicate terms can be affirmed.
- The identity of the subject and predicate terms can be denied.
- Complex terms can be negated (Ladd identifies three ways: Boole/Jevons; DeMorgan; Schröder).

Categorical propositions in algebra (1)

We can either

> *assign the expression of the 'quantity' of propositions to the copula or to the subject [Ladd, 1883, p. 23].*

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- If quantity is assigned to the copula, then two copulas are necessary (one universal, one partial). McColl, Peirce
- If it is assigned to the subject term, the only copula that is needed is identity. Boole, Schröder, Jevons

Categorical propositions in algebra (2)

	Traditional	Boole / Schröder	Jevons	McColl	Peirce
Uni-	All a is b	$a = vb$	$a = ab$	$a : b$	$a \prec b$
versal	No a is b	$a = v\bar{b}$	$a = a\bar{b}$	$a : \bar{b}$	$a \prec \bar{b}$
Part-	Some a is b	$va = vb$	$ca = cab$	$a \div \bar{b}$	$a \overline{\prec} \bar{b}$
ial	Some a is not b	$va = v\bar{b}$	$ca = ca\bar{b}$	$a \div b$	$a \overline{\prec} b$

In the Boole/Schröder approach, v is not a categorical term like a or b, but a special term that picks out an arbitrary indefinite class. Jevons's c works similarly, but he does not distinguish it in the way that v is distinguished; it can be any other class term [Ladd, 1883, p. 24].

Which way is preferable?

Advantages of the second way (Boole/Schröder/Jevons):

- Only one copula is necessary.

Advantages of the first way (McColl/Peirce):

- Copulas that include their quantity can be used to link either terms *or* propositions, so that, e.g., $a \overline{\prec} b$ can be read either "a is not wholly contained under b" or "a does not imply b" [Ladd, 1883, p. 24].
- There is a correspondence between the quantity of the copula and its quality. The universal copulas are positive (affirmative), and the partial copulas are negative [Ladd, 1883, p. 25].

Ladd's way

Instead of taking as basic:

(a)	$a \prec b$	"a is wholly b"
(o)	$a \overline{\prec} b$	"a is not wholly b

take:

(e)	$a \overline{\vee} b$	"a is-wholly-not b"
(i)	$a \vee b$	"a is-partly b"

Symmetry

Both $\vee$ and $\overline{\vee}$ are symmetric combinators: "the propositions $a \,\overline{\vee}\, b$,$a \vee b$, may be read either forward or backward" [Ladd, 1883, p. 26]; inclusion statements using $\prec$ are asymmetric.

Inclusions can be converted into exclusions by changing the copula and the sign of the predicate [Ladd, 1883, p. 27]:

$$a \prec b = a \,\overline{\vee}\, \bar{b}$$

Every exclusion is equivalent to a pair of inclusions, differing on which of the two terms you take as the predicate:

$$\begin{array}{ccccc} a \,\overline{\vee}\, b & = & a \prec \bar{b} & = & b \prec \bar{a} \\ a \vee b & = & a \overline{\prec} \bar{b} & = & b \overline{\prec} \bar{a} \end{array}$$

Advantages of this approach

- Categoricals and hypotheticals are treated identically in the formal system [Ladd, 1883, p. 23].
- If we let p denote a premise and c a conclusion following from p, then we can express this consequent fact as either:

$$p \mathbin{\overline{\vee}} \bar{c}$$

or

$$\bar{c} \mathbin{\overline{\vee}} p$$

Existence and nonexistence claims

We can express existence and nonexistence claims via ∞:

$$x \mathbin{\overline{\vee}} \infty \tag{1}$$

means "x does not, under any circumstances, exist", and

$$x \vee \infty \tag{2}$$

means that "x is at least sometimes existent" [Ladd, 1883, p. 29].

Translating between propositions and terms

Unlike in the (pure) algebra of terms, the algebra of propositions does not have 0 [Ladd, 1883, p. 29]. As a result, we can drop reference to ∞ in contexts where it can be restored without ambiguity. Therefore, we can write (1) and (2) as:

$$x\overline{\bigtriangledown} \tag{1'}$$

and

$$x\vee \tag{2'}$$

This notation allows us to translate from categorical propositions (e.g., $a \,\overline{\bigtriangledown}\, b$ "No a is b") into statements about terms (e.g., $ab\overline{\bigtriangledown}$ "The combination ab does not exist") [Ladd, 1883, p. 30].

On the elimination of terms

> *The most common object in reasoning is to eliminate a single term at a time—namely, one which occurs in both premises [Ladd, 1883, p. 37].*

This goal of logic can be accomplished via the inference form "if a is b and c is d, then ac is bd [Ladd, 1883, p. 34]:

$$(a \,\overline{\vee}\, b)(c \,\overline{\vee}\, d) \,\overline{\vee}\, (ac \vee b + d) \tag{I}$$

By setting d equal to $\bar{b}$, so that $b + d = \infty$, we can rewrite (I) as:

$$(a \,\overline{\vee}\, b)(\bar{b} \,\overline{\vee}\, c)(c \vee a)\overline{\vee} \tag{II}$$

(going via the intermediate equation $(a \,\overline{\vee}\, b)(c \,\overline{\vee}\, \bar{b})(ac \vee \infty)\overline{\vee}$).

The main result (again)

Theorem

The argument of inconsistency,

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is the single form to which all the ninety-six valid syllogisms (both universal and particular) may be reduced [Ladd, 1883, p. 40].

Proof.

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Rules for the validity of a syllogism

From this theorem a corollary follows, in the form of an easy to apply rule, stated in ordinary English, for identifying whether any syllogism is valid:

Rule

Take the contradictory of the conclusion, and see that the universal propositions are expressed with a negative copula and particular propositions with an affirmative copula. If two of the propositions are universal and the other particular, and if that term only which is common to the two universal propositions has unlike signs, then, and only then, the syllogism is valid [Ladd, 1883, p. 41].

An example

Example

The syllogism Baroco:

$$\begin{gathered}\text{All } P \text{ is } M \\ \text{Some } S \text{ is not } M \\ \therefore \text{Some } S \text{ is not } P\end{gathered}$$

is equivalent to the inconsistency

$$(P \mathbin{\overline{\triangledown}} \bar{M})(S \vee \bar{M})(S \mathbin{\overline{\triangledown}} P)\overline{\triangledown}$$

Aristotle already knew how to do this

Aristotle reduced both

Baroco	**Bocardo**
All P are M	Some M is not P
Some S is not M	All M are S
$\therefore$ Some S is not P	$\therefore$ Some S is not P

to

Barbara
All M are P
All S are M
$\therefore$ All S are P

via *reductio ad absurdem*—that is, taking the contradictory of the conclusion and replacing one of the premises with it, and then making the contradictory of the replaced premise the conclusion.

How what Ladd does differs

However, instead of taking Barbara as basic and Baroco and Bocardo as derived, Ladd showed that one can take as "basic" the inconsistency of the following three claims:

All M are P
All S are M
Not all S are P

Any two of these propositions entails the denial of the third; which is to say that the contradictory of any of the propositions follows from the other two, which gives us all three syllogisms.

This is the sense in which the three syllogisms are reduced to one "form".

What's the benefit of doing this? (1)

First, Ladd says:

> *If for the usual three statements consisting of two premises and a conclusion one substitutes the equivalent three statements that are together incompatible. . . one has a formula which has this great advantage: the order of the statements is immaterial—the relation is a perfectly symmetrical one [Ladd-Franklin, 1928, p. 532].*

In addition to the symmetry of the relation, the result is

> *a source of great simplicity—there is only one valid form of the antilogism instead of the fifteen valid forms of the syllogism which common logic requires us to bear in mind [Ladd, 1883, p. 532].*

What's the benefit of doing this? (2)

Thirdly, both the simplicity and the symmetry can be improved upon if all of the three claims can be written as either (e) "No S are P" or (i) "Some S is P" claims, which can be simply converted.

If we admit 'infinite' terms, then this rewriting into symmetric propositions is always possible, as "All S are non-P" is equivalent to "No S is P", and "Some S is not P" is equivalent to "Some S is non-P".

Solving the Inverse Logical Problem (3)

Ladd's antilogism gives rise to a rule which gives a systematic method for determining conditions inconsistent with certain combinations. Because any given syllogism can be reduced to a syllogism of the form given in Theorem 0.1, one need not blindly invent laws and see if they agree with results.

A bit of historiography (1)

In the earliest review of Ladd's dissertation, [Anonymous, 1883], no specific mention is made of this result. The (unidentified) reviewer introduces Ladd's new notation, $\vee$ and $\overline{\vee}$, gives its semantics and formation rules, and notes that "with these she is able to write algebraically all the old forms of statement, and to perform the customary operations of symbolic logic with great brevity and facility" [Anonymous, 1883, p. 514].

The singling out of the antilogism as a fundamental contribution is first (as far as I can tell) made by Brown: "when Mrs. Ladd-Franklin has demonstrated that one simple form underlies all syllogism..." [Brown, 1909, p. 304].

A bit of historiography (2)

Shen quotes "the late Professor Josiah Royce of Harvard":

> *There is no reason why this should not be accepted as the definitive solution of the problem of the reduction of syllogisms. It is rather remarkable that the crowning activity in a field worked over since the days of Aristotle should be the achievement of an American woman" [Shen, 1927, p. 60].*

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> *In a newspaper clip "To Get Her Degree Earned Years Ago", Josiah Royce is quoted as describing her thesis work as "the crowning activity in a field worked over since the days of Aristotle". "The [Aristotelian] system was never fully demonstrated until Mrs. Ladd-Franklin worked out the whole method at Johns Hopkins" (The Hartford Courant, February 21, 1926, p. 20) [Pietarinen, 2013, fn. 6].*

Concluding remarks

- In her 1883 dissertation, Ladd-Franklin introduced to Boolean algebra a pair of symmetric copula.
- This allowed her to define the "antilogism", an "inconsistent triad" that could be used to represent every valid syllogism.
- People recognised the utility of this representation soon after her work.
- Within 30 years, people made the leap to her formula being a solution to a problem.
- Within 40 years, people attributed the problem to Aristotle.
- At some point after that, the problem attributed to Aristotle was attributed as a problem to all intervening logicians, too.
- While she solved a problem, it certainly wasn't Aristotle's, nor had it vexed people for millennia.
- Instead, it was a problem due to Jevons, that both Schröder and Boole had attempted to solve [Uckelman, 2021].

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Thank You!